STAT354 Exam 2007

Question 1 (25 marks)

The two independent random variables X and Y are both distributed $U(0, 1)$.

(a) Use the change of variable method to find the distribution of $(X - Y)^2$. *[20 marks]*

(b) Explain the similarity of the distribution in (a) to the distribution of a χ^2_1. *[5 marks]*

Question 2 (25 marks)

(a) For $\boldsymbol{x} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, show that any linear combination $\boldsymbol{a}'\boldsymbol{x} \sim N_1(\boldsymbol{a}'\boldsymbol{\mu}, \boldsymbol{a}'\boldsymbol{\Sigma}\boldsymbol{a})$. *[12 marks]*

(b) Show that if $\boldsymbol{a}'\boldsymbol{x} \sim N_1(\boldsymbol{a}'\boldsymbol{\mu}, \boldsymbol{a}'\boldsymbol{\Sigma}\boldsymbol{a})$ for *every* $\boldsymbol{a}$ then $\boldsymbol{x}$ *must* be $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. *[13 marks]*

Question 3 (25 marks)

Sampling is undertaken from a population with density function

$$f(x) = \frac{1}{2}\theta^3 x^2 e^{-\theta x}, \; x > 0$$

(a) Does θ possess a sufficient statistic? Show all working. *[8 marks]*

(b) Determine the minimum variance bound for an unbiased estimate of θ. *[9 marks]*

(c) Will an unbiased estimator for θ necessarily exist that attains the bound? Give full reasons. *[8 marks]*

Question 4 (25 marks)

(a) Determine the posterior distribution for the population parameter assuming a sample of size n from the Poisson distribution, using the approximation based on the likelihood. *[10 marks]*

(b) What is the posterior distribution based on a conjugate prior? *[15 marks]*

STA354 : FORMULAE

3.
(a)

$$f(\boldsymbol{X};\theta) = g[t(\boldsymbol{X};\theta)]h(\boldsymbol{X})$$

(b)

$$V(T) \geq \frac{1}{I_{\boldsymbol{X}}(\theta)} \ ; \ I_{\boldsymbol{X}}(\theta) = E\left(\frac{\partial \log L(\theta)}{\partial \theta}\right)^2 = -E\left(\frac{\partial^2 \log L(\theta)}{\partial \theta^2}\right)^2 \ , \ E(T) = \theta$$

4.
(a)

$$L(\theta|\boldsymbol{x}) \propto N(\widehat{\theta}, 1/I(\widehat{\theta}|\boldsymbol{x}))$$

(b)

$$f(x;\theta) = p(x|\theta) = B(\theta)h(x)e^{q(\theta)K(x)}$$

$$p(\theta) \propto B(\theta)e^{q(\theta)\tau}$$

$$p(\theta|\boldsymbol{x}) \propto B^{n+1}(\theta)e^{q(\theta)(\tau + \sum_i K(x_i)}$$