STAT354 Exam 2006

Question 1

Two geometric random variables Y_1 and Y_2 are independently and identically distributed with probability function

$$P(Y = y) = p(1-p)^{y-1},\ y = 1, 2, \ldots$$

where y is the number of Bernoulli trials to the first success. The probability of success at each trial is p.

(a) Produce a plot of the space (Y_1, Y_2).

(b) Using the change of variable method, find the distribution of $Y_T = Y_1 + Y_2$.
Show all working.

(c) Describe the distribution of Y_T.

(d) Hence, or otherwise, give $E(Y_T)$. Show all working.

Question 2

(a) Derive the pdf for the smallest order statistic for a sample of size n.

(b) Links in a chain have a distribution of breaking strain (Y) given by

$$f(y) = \frac{1}{\mu} e^{-y/\mu},\quad y, \mu > 0$$

If a chain is composed of 100 links, find the density of the breaking strain of the chain.

(c) Compare the density for breaking strain for the chain with the density of breaking strain for the links and comment.

(d) In an attempt to improve chain quality, the breaking strain of individual links is to be increased, at a cost. Comment on the effect(s) of such a change.

Question 3

Sampling is undertaken from a population with density function

$$f(x) = \theta^2 x e^{-\theta x}$$

(a) Is the sampling from an exponential family of distributions? Give full reasoning.

(b) Determine the minimum variance bound for an estimate of θ.

(c) Will an unbiased estimator for θ necessarily exist that attains the bound?

Give full reasons.

(d) Does θ possess a sufficient statistic? Show all working.

Question 4

(a) A simple experiment is constructed to test the difference between proportions from a control and a single treatment. The data for control consists of x_c successes out of n_c Bernoulli trials, while the treatment yielded x_t successes in n_t similar trials.

Construct a likelihood ratio test for this experiment, giving a full outline of all the main points.

(Do not attempt to simplify the likelihood ratio)

(b) (i) Construct a conjugate prior for the Bernuolli distribution.

(ii) What is Jeffrey's prior for the Bernoulli distribution? Show all working.

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2.
(a)

$$f_{Y_1}(y_1) = n\,[1 - F(y_1)]^{n-1}\, f(y_1),\ -\infty < y_1 < \infty$$

3.
(a)

$$f(x;\theta) = B(\theta)h(x)e^{p(\theta)K(x)}$$

(b)

$$V(T) \geq \frac{1}{I_{\boldsymbol{X}}(\theta)}\ ;\ I_{\boldsymbol{X}}(\theta) = E\left(\frac{\partial \log L(\theta)}{\partial\theta}\right)^2 = -E\left(\frac{\partial^2 \log L(\theta)}{\partial\theta^2}\right)^2\ ,\ E(T) = \theta$$

$$V(T') \geq \frac{[\tau'(\theta)]^2}{I_{\boldsymbol{X}}(\theta)}\ ,\ E[T'(\theta)] = \tau(\theta)$$

(d)

$$f(\boldsymbol{X};\theta) = g[t(\boldsymbol{X};\theta)]h(\boldsymbol{X})$$

4.
(b)

$$P(Y = y) = \binom{1}{y}\pi^y(1-\pi)^{1-y},\ y = 0, 1$$

(i)

$$f(x;\theta) = p(x|\theta) = B(\theta)h(x)e^{q(\theta)K(x)}$$

$$p(\theta) \propto B(\theta)e^{q(\theta)\tau}$$

(ii)

$$p(\theta) \propto \sqrt{I(\theta|x)}$$