STAT354 Exam A2005

Question 1

Two independent random variables X and Y are exponentially distributed, viz,

$$f(x) = \frac{e^{-x/\alpha}}{\alpha}, \; x > 0, \text{ and, } f(y) = \frac{e^{-y/\beta}}{\beta}, \; y > 0.$$

(a) Use the change of variable method to find the distribution of $U = X + Y$.

Show all working.

(b) Check your solution to (a) using the marginal distribution.

(c) Briefly examine the case $\alpha = \beta$.

Question 2

The median is calculated for a sample size of n, where n is an odd integer.

(a) Give the distribution function for the sample median.

(b) One of the simple trend estimation methods used in the early applications of spatial statistics was the `spatial moving average`. This measure used the average of the r nearest neighbours, where r was small in comparison to the size of the data set. An alternative would be to use a `spatial moving median`, the median of the r nearest neighbours, especially if the data contained outliers.

Give the distribution function for such a spatial moving median based on the 3 nearest neighbours, assuming that the data are uniformly distributed $U(0, 1)$ as the result of coordinate scaling.

(c) Is the spatial moving median a genuine trend estimation method? Give reasons.

(d) Briefly outline the extension of (b) to the general case of r nearest neighbours.

Question 3

A random sample of size n is taken from a population with distribution fuction

$$f(y;\theta) = (\theta+1)y^{\theta},\ 0 < y < 1,\ \theta > 0.$$

(a) Does this distribution belong to the exponential family? Show working.

(b) Does a sufficient statistic for θ exist? Give reasons.

(b) Show that the minimum variance bound for an estimator of $1/(1+\theta)$ is independent of θ.

Question 4

A random sample $X_1, \ldots, X_n$ is such that $X_i \sim N(\mu, \sigma_i^2)$ where the σ_i^2 are known.

(a) Derive a likelihood ratio test for testing

$$H_0 : \mu = \mu_0 \ \ vs \ \ H_1 : \mu \neq \mu_0$$

(b) Verify your solution to (a) for the case $\sigma_i^2 = \sigma^2 \ \forall\ i$.

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2.
(a)

$$f_{Y_r}(y_r) = \frac{n!}{(n-r)!(r-1)!}\,[F(y_r)]^{r-1}\,[1-F(y_r)]^{n-r}\,f(y_r),\ -\infty < y_r < \infty$$

3.
(a)

$$f(y;\theta) = B(\theta)h(x)e^{p(\theta)K(x)}$$

(b)

$$f(\boldsymbol{X};\theta) = g[t(\boldsymbol{X};\theta)]h(\boldsymbol{X})$$

(c)

$$V(T) \geq \frac{1}{I_{\boldsymbol{X}}(\theta)}\ ;\ I_{\boldsymbol{X}}(\theta) = E\left(\frac{\partial \log L(\theta)}{\partial\theta}\right)^2 = -E\left(\frac{\partial^2 \log L(\theta)}{\partial\theta^2}\right)^2\ ,\ E(T) = \theta$$

$$V(T') \geq \frac{[\tau'(\theta)]^2}{I_{\boldsymbol{X}}(\theta)}\ ,\ E[T'(\theta)] = \tau(\theta)$$