STAT354 Exam 2004

Question 1

Two random variables X and Y have the distribution function

$$f(x, y) = 2,\ 0 < x < y < 1$$

(a) Verify that the given function is a distribution function.

(b) Obtain the marginal distributions for X and Y.

(c) Using the change of variable technique, obtain the distribution of $X + Y$.

(d) Use the results from (b) to verify your results in (c), using marginal distributions.

Question 2

Let $Y_1, \ldots, Y_n$ be a random sample from a distribution with df

$$f(y; \alpha) = 2\alpha^4 y^3 e^{-\alpha^2 y^2},\ \ \alpha > 0,\ y > 0.$$

(a) Derive the minimum variance bound for an unbiased estimator of α. You are given that $E(Y^2) = 2/\alpha^2$.

(b) Show that the minimum variance bound of an unbiased estimator of $\log \alpha$ is independent of α.

Question 3

(a) The random variable Y follows the lognormal distribution if

$$\log Y \sim N(\mu, \sigma^2).$$

The distribution function of Y is

$$f(y; \mu, \sigma^2) = \frac{1}{\sigma y \sqrt{2\pi}} e^{-(\log y - \mu)^2 / 2\sigma^2}, \quad 0 < y < \infty.$$

A random sample of size n is chosen from such a distribution.

(i) Determine the maximum likelihood estimators for μ and σ^2 and the covariance matrix of these estimators.

(ii) Compare the estimators obtained in (i) with those obtained by random sampling from a normal distribution.

(b) For a random sample $Y_1, \ldots, Y_n$ from a population with df

$$f(y; \theta) = \theta 2^{\theta} / y^{1+\theta}, \quad 2 < y < \infty, \ \theta > 0,$$

(i) determine whether a sufficient estimator exists for θ.

(ii) If so, construct such an estimator.

Question 4

Two large batches of logs are offered for sale to a mining company whose concern is to have the diameters as uniform as possible. Batch A is more expensive than batch B, but the extra cost will be offset by the uniformity of product. Thus batch A would be preferred if the standard deviation of diameters in batch A is less than 1/2 the standard deviation of the diameters in batch B.

Produce a likelihood ratio test for deciding which batch should be purchased based on the results of samples of the same size from each batch.

STA354 : FORMULAE

2.

$$V(T) \geq \frac{1}{nI(\theta)}$$

3.
(a)

$$I_{ij}(\theta) = -E\left[\frac{\partial^2 \log L(\theta)}{\partial\theta_i \partial_j \theta_j}\right]$$

(b)

$$\Pi_i f(y_i;\theta) = g(t;\theta)s(y_1,\ldots,y_n)$$