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PMTH212, Multivariable Calculus

Assignment Summary – 2007

For External Students

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This assignment summary may be detached for your notice board.
Textbook and Lecture Notes

The textbook is Calculus by H. Anton, 6th or 7th edition, John Wiley & Sons.

The lecture notes you have received cover all the material required of this unit. It is essential that you read carefully these lecture notes and try to understand everything presented there. While the booklet of lecture notes is more concise, the above mentioned textbook contains more material and also many exercise questions. Therefore the lecture notes and the textbook can be used to complement each other.

The following textbooks are recommended as reference books, where you can find slightly different ways of presenting the topics covered by this unit and also some more material related to this unit.


Calculus of Several Variables by R.A. Adams, 2nd edition, Addison-Wesley Publishers Ltd.

Assessment

There will be a three-hour examination in the June-July examination period. The final assessment consists of two parts: your assignment solutions will be counted as 20% and the June-July examination will be counted as 80%.

Lectures

Three lectures per week during the First Semester. Past experience shows that those internal students who do not come to most of the lectures usually fail the unit at the end. Therefore, if you are an internal student, I strongly urge you to attend the lectures.

Tutorials

One tutorial per week mainly to help internal students with their assignments.

Assignments

Assignments are an essential part of this unit. It is important to notice that through doing the assignments and through the feedbacks from your marked assignments you
can check your understanding of the material in the unit in a timely fashion. As poor understanding of earlier material generally makes the study of the later part of the unit much more difficult, I urge you try not to get too far behind, and not to leave important material unfinished before going to the next step. It is important that you submit your assignments at the scheduled times. Late assignments, especially towards the end of the semester, may not be marked.

If you are stuck with a particular question in your assignment, try to get help. I’m just too happy to be able to help you with any of your questions. If after considerable efforts, you still cannot do it, then it is better to submit your assignment without answering that particular question than having the assignment rather late. You will receive complete solutions to the assignment problems when the marked assignments are returned.

If you are external, your assignments should go through the Teaching and Learning Center so as to be recorded as received by the University.

Unit Outline

This unit is a logical extension of the calculus studied in the first-year mathematics units. It mainly consists of Chapters 12–16 of Anton, 7th edition (Chapters 13-17 in the 6th edition). The material for Lecture 17, *Parametric Problems*, is not covered by Anton. The specific topics covered by this unit are listed in the timetable on pages 3-4 below.

You might find it helpful to have your first year calculus notes handy, as the first year material is frequently used in this unit. In particular, if you find yourself rather rusty with the first year differentiation and integration techniques, it might be a good idea to have a review of them before you engage in working on the partial derivatives and multiple integrals in this unit.

Instructions for Examination

The examination questions will be concerned with basic concepts and general methods. The style will be similar to the examples in the lecture notes, and the questions in the assignments and the past examination papers. At the end of this study guide, you can find the 1997 and 2001 exam papers and their solutions.

Unit Coordinator

If you have any specific questions concerning the work, or the unit in general, please don’t hesitate to contact me. You can reach me by phone on: (02)6773 3154 or email syan@turing.une.edu.au. You may fax if necessary on (02)6773 3312. My office is B163. You are most welcome to come to see me on matters related to this unit.

Wish you enjoyment and success in your studies.

Dr. Shusen Yan
Unit Coordinator
Submission dates for external assignments are given in the Assignment Summary page. For internal students, Wednesday is the due day for the assignment based on the material taught in the previous week. The section numbers referred to below are based on the 7th edition of Anton, followed by the numbers of the corresponding material in the 6th edition in brackets.

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Assignment 12
ASSIGNMENT 1

1. Show that the points (2, 1, 6), (4, 7, 9) and (8, 5, −6) are the vertices of a right triangle.

2. Find the distance from the point (5, 2, 3) to the
   (a) $xy$-plane  (b) $xz$-plane  (c) $y$-axis.

3. Find the terminal point of $v = i + 2j - 3k$ if the initial point is $(-2, 1, 4)$.

4. Find a vector $n$ which is perpendicular to the plane determined by the points $A(0, -2, 1)$, $B(1, -1, -2)$, and $C(-1, 1, 0)$.

5. Find parametric equations for the line
   (a) in $R^2$ through (1,1) and parallel to the line $x = -5 + t, \ y = 1 - 2t$;
   (b) in $R^3$ through $(x_0, y_0, z_0)$ and $(x_1, y_1, z_1)$.

6. Determine whether the planes are perpendicular.
   (a) $x - y + 3z - 2 = 0, \ 2x + z = 1$
   (b) $3x - 2y + z = 1, \ 4x + 5y - 2z = 4$.

7. Find equations of the planes.
   (a) Through $(-1, 4, -3)$ and perpendicular to the line $x - 2 = t, \ y + 3 = 2t, \ z = -t$
   (b) Through $(-1, 2, -5)$ and perpendicular to the planes $2x - y + z = 1$ and $x + y - 2z = 3$. 
ASSIGNMENT 2

1. Name and sketch the surfaces:
   (a) \((x - 1)^2/4 + (y - 2)^2/9 + (z - 4)^2/16 = 1\)
   (b) \(z^2 = 4x^2 + y^2 + 8x - 2y + 4z\)

2. Find an equation of the orthogonal projection onto the xy-plane of the curve of intersection of the surfaces.
   (a) The paraboloids \(z = x^2 + y^2\) and \(z = 1 - 4x^2 - y^2\).
   (b) The paraboloid \(z = 4 - x^2 - y^2\) and the parabolic cylinder \(z = y^2\)

3. Describe the graph of the vector equation.
   (a) \(r = (3 \sin 2t)i + (3 \cos 2t)j\)  
   (b) \(r = -2i + tj + (t^2 - 1)k\)

4. Determine whether \(r = \cos(t^2)i + \sin(t^2)j + e^{-t}k\)
   is a smooth curve of the parameter \(t\).

5. Let \(u, v, w\) be differentiable vector-valued functions of \(t\). Prove that
   \[
   \frac{d}{dt} [u \cdot (v \times w)] = \frac{du}{dt} \cdot [v \times w] + u \cdot \left[ \frac{dv}{dt} \times w \right] + u \cdot [v \times \frac{dw}{dt}]
   \]

6. (a) Evaluate \(\int [(t \sin t)i + j] dt\);
   (b) Find the arc length of the curve \(r(t) = (3 \cos t)i + (3 \sin t)j + tk; \ 0 \leq t \leq 2\pi\).
ASSIGNMENT 3

1. Find the unit tangent vector $T$ and the unit normal vector $N$ for the given value of $t$.
   (a) $x = e^t$, $y = e^{-t}$, $z = t$; $t = 0$.
   (b) $x = e^t \cos t$, $y = e^t \sin t$, $z = e^t$; $t = 0$.

2. Find the curvature at the indicated point.
   (a) $\mathbf{r}(t) = (4 \cos t)\mathbf{i} + (\sin t)\mathbf{j}$; $t = \pi/2$.
   (b) $x = e^t, y = e^{-t}, z = t$; $t = 0$.

3. Show that for a plane curve described by $y = f(x)$ the curvature $\kappa(x)$ is

\[ \kappa(x) = \frac{|d^2y/dx^2|}{[1 + (dy/dx)^2]^{3/2}} \]

[Hint: Let $x$ be the parameter so that $\mathbf{r}(x) = x\mathbf{i} + y\mathbf{j} = x\mathbf{i} + f(x)\mathbf{j}$.]

4. Let $f(x, y) = x + (xy)^{1/3}$. Find
   (a) $f(t, t^2)$, (b) $f(x, x^2)$, (c) $f(2y^2, 4y)$.

5. Find $g(u(x, y), v(x, y))$ if $g(x, y) = y \sin(x^2y)$, $u(x, y) = x^2y^3$, $v(x, y) = \pi xy$.

6. Let $f(x, y, z) = zxy + x$. Find
   (a) $f(x + y, x - y, x^2)$, (b) $f(xy, y/x, xz)$.

7. Describe the level surfaces.
   (a) $f(x, y, z) = 3x - y + 2z$  (b) $f(x, y, z) = z - x^2 - y^2$. 

ASSIGNMENT 4

1. Find the region where the function $f$ is continuous.
   (a) $f(x, y) = (x - y)^{1/2}$.
   (b) $f(x, y) = \ln(2x - y + 1)$.
   (c) $f(x, y) = \cos\left(\frac{xy}{1 + x^2 + y^2}\right)$.

2. Find the limit, if it exists.
   (a) $\lim_{(x,y) \to (0,0)} \frac{x^4 - 16y^4}{x^2 + 4y^2}$.
   (b) $\lim_{(x,y) \to (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$.

3. (a) Show that the value of $\frac{x^3 y}{2x^6 + y^2}$ approaches 0 as $(x, y) \to (0, 0)$ along any straight line $y = mx$, or along any parabola $y = kx^2$.
   (b) Show that $\lim_{(x,y) \to (0,0)} \frac{x^3 y}{2x^6 + y^2}$ does not exist.
   [Hint: Let $(x, y) \to (0, 0)$ along the curve $y = x^3$, and then compare the result with the results in (a).]

4. Find $f_x$, $f_y$ and $f_z$.
   (a) $f(x, y, z) = z \ln(x^2 y \cos z)$.  (b) $f(x, y, z) = (\frac{xz}{1 - z^2 - y^2})^{-3/4}$.

5. (a) Show that $f(x, y) = \ln(x^2 + y^2)$ satisfies
   $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.
   This is called the Laplace’s Equation.
   (b) Show that $u(x, y) = e^x \cos y$ and $v(x, y) = e^x \sin y$ satisfy
   $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$,  $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.
   These are called the Cauchy-Riemann Equations.

6. Let $f(x, y) = (x^2 + y^2)^{2/3}$. Show that $f_x(0, 0) = 0$. 
ASSIGNMENT 5

1. 
   (a) Let \( f(x, y) = \frac{x^3}{y} \). Find \( f_{xy} \) and \( f_{yx} \) and verify their equality.
   (b) Let \( f(x, y) = e^{xy^2} \). Find \( f_{xyx} \), \( f_{xxy} \) and \( f_{yxx} \) and verify their equality.

2. Let \( z = 3x - 2y \), where \( x = u + v \ln u, y = u^2 - v \ln v \). Find \( \partial z/\partial v \) and \( \partial z/\partial u \) by the chain rule.

3. Show that if \( u(x, y) \) and \( v(x, y) \) satisfy the Cauchy-Riemann equations (See Assignment 4, Question 5), and if \( x = r \cos \theta \) and \( y = r \sin \theta \), then
   \[
   \frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}.
   \]

4. Find equations for the tangent plane and normal line to the given surfaces at the point \( P \).
   (a) \( z = (1/2)x^3y^{-2} \); \( P(2, 4, 4) \).
   (b) \( z = \ln[(x^2 + y^2)^{1/2}] \); \( P(-1, 0, 0) \).

5. The volume \( V \) of a right-circular cone of radius \( r \) and height \( h \) is given by \( V = (1/3)\pi r^2 h \). Suppose the height decreases from 20in. to 19.95in., while the radius increases from 4in. to 4.05in. Use a total differential to approximate the change in volume.

6. 
   (a) Let \( f(x, y) = y/(x + y) \). Find a unit vector \( \mathbf{u} \) for which \( D_{\mathbf{u}}f(2, 3) = 0 \).
   (b) Find a unit vector \( \mathbf{u} \) that is perpendicular at \( P(2, -3) \) to the level curve of \( f(x, y) = 3x^2y - xy \) through \( P \).
ASSIGNMENT 6

1. Find a unit vector in the direction in which \( f(x, y, z) = (x - 3y + 4z)^{1/2} \) increases most rapidly at \( P(0, -3, 0) \), and find the rate of increase of \( f \) in that direction.

2. Two surfaces are said to be orthogonal at a point of intersection if their normal lines are perpendicular at that point. Prove that the surfaces \( f(x, y, z) = 0 \) and \( g(x, y, z) = 0 \) are orthogonal at a point of intersection, \( (x_0, y_0, z_0) \), if and only if

\[
fxgy + gygx + fzgz = 0
\]

at \( (x_0, y_0, z_0) \). Here we suppose \( \nabla f(x_0, y_0, z_0) \neq 0 \) and \( \nabla g(x_0, y_0, z_0) \neq 0 \).

3. Let \( f \) be a function of one variable and let

\[
z = f(x^2 + y^2)\]

Show that \( y \partial z / \partial x - x \partial z / \partial y = 0 \).

4. Assume that \( F(x, y, z) = 0 \) defines \( z \) implicitly as a function of \( x \) and \( y \). Show that

\[
\partial z / \partial x = -\frac{\partial F / \partial x}{\partial F / \partial z}, \quad \partial z / \partial y = -\frac{\partial F / \partial y}{\partial F / \partial z}.
\]

5. Let \( F(u, v) = \int_u^v f(t) \, dt \). Show that

(a) \( \partial F / \partial u = -f(u), \ \partial F / \partial v = f(v) \);

(b) \( \frac{d}{dx} F(a(x), b(x)) = \frac{d}{dx} \int_{a(x)}^{b(x)} f(t) \, dt = f(b(x))b'(x) - f(a(x))a'(x) \).
ASSIGNMENT 7

1. Locate all relative maxima, relative minima and saddle points.
   (a) \( f(x, y) = x^2 + xy - 2y - 2x + 1. \)
   (b) \( f(x, y) = 2x^2 - 4xy + y^4 + 2. \)

2. Find the absolute extreme of the given function on the indicated set R.
   (a) \( f(x, y) = xy - 2x; \) R is the triangular region with vertices (0,0), (0,4) and (4,0).
   (b) \( f(x, y) = xy^2; \) R is the region that satisfies \( x \geq 0, y \geq 0, x^2 + y^2 \leq 1. \)

3. Use Lagrange multipliers to find the maximum and minimum values of \( f \) subject to the given constraint.
   (a) \( f(x, y) = x^2 - y; \) \( x^2 + y^2 = 25. \)
   (b) \( f(x, y) = x - 3y - 1; \) \( x^2 + 3y^2 = 16. \)

4. Find the point on the line \( 2x - 4y = 3 \) that is closest to the origin.

5. Find the points on the surface \( xy - z^2 = 1 \) that are closest to the origin.

6. Given \( F(x) = \int_0^1 t^x dt = (1 + x)^{-1} \) for \( x > -1. \) By repeated differentiation of this identity, evaluate the integral \( \int_0^1 t^x (\ln t)^n dt. \)
ASSIGNMENT 8

1. Evaluate the iterated integral.
   (a) \( \int_0^{\pi/2} \int_0^{\sin y} e^x \cos y \, dx \, dy \).
   (b) \( \int_0^1 \int_0^x e^{x^2} \, dy \, dx \).

2. Evaluate the double integral.
   (a) \( \int_R (x + y) \, dA \); \( R \) is the region enclosed between the curves \( y = x^2 \) and \( y = x^{1/2} \).
   (b) \( \int_R x \cos y \, dA \); \( R \) is the triangular region bounded by \( y=x \), \( y=0 \) and \( x=\pi \).

3. Use double integral to find the volume of the solid in the first octant bounded above by the paraboloid \( z = x^2 + 3y^2 \), below by the plane \( z = 0 \) and laterally by \( y = x^2 \) and \( y = x \).

4. Evaluate the integral by first reversing the order of integration.
   (a) \( \int_0^2 \int_{g/2}^1 \cos(x^2) \, dx \, dy \).
   (b) \( \int_1^3 \int_0^{\ln x} x \, dy \, dx \).

5. (a) Use polar coordinates to evaluate \( \int_R (9 - x^2 - y^2)^{1/2} \, dA \), where \( R \) is the region in the first quadrant within the circle \( x^2 + y^2 = 9 \).
   (b) Evaluate the iterated integral by converting to polar coordinates.
   \( \int_{-2}^2 \int_{-(4-y^2)^{1/2}} f(4-y^2)^{1/2} e^{-(x^2+y^2)} \, dx \, dy \).

6. (a) Find the surface area of the portion of the plane \( 2x + 2y + z = 8 \) in the first octant that is cut off by the three coordinate planes.
   (b) Find the surface area of the portion of the paraboloid \( 2z = x^2 + y^2 \) that is inside the cylinder \( x^2 + y^2 = 8 \).
ASSIGNMENT 9

1. Evaluate the triple integral \( \iiint_{R} \cos(z/y) \, dV \), where \( R \) is defined by
   \[ \frac{\pi}{6} \leq y \leq \frac{\pi}{2}, \quad y \leq x \leq \frac{\pi}{2}, \quad 0 \leq z \leq xy. \]

2. Express the integral as an equivalent integral in which the \( z \)-integration is performed first, the \( y \)-integration second and the \( x \)-integration last.
   (a) \( \int_{0}^{3} \int_{0}^{(9-z^2)^{1/2}} \int_{0}^{(9-y^2-z^2)^{1/2}} f(x, y, z) \, dx \, dy \, dz \).
   (b) \( \int_{0}^{4} \int_{0}^{x/2} \int_{0} f(x, y, z) \, dy \, dz \, dx \).

3. Let \( G \) be the rectangular box defined by \( a \leq x \leq b, c \leq y \leq d, k \leq z \leq l \). Show that
   \[ \iiint_{G} f(x)g(y)h(z) \, dV = \left[ \int_{a}^{b} f(x) \, dx \right] \left[ \int_{c}^{d} g(y) \, dy \right] \left[ \int_{k}^{l} h(z) \, dz \right]. \]

4. (a) Use cylindrical coordinates to find the volume of the solid bounded above and below by the sphere \( x^2 + y^2 + z^2 = 9 \) and laterally by the cylinder \( x^2 + y^2 = 4 \).
   (b) Use spherical coordinates to find the volume of the solid bounded above by the sphere \( \rho = 4 \) and below by the cone \( \phi = \pi/3 \).

5. Let \( G \) be the solid in the first octant bounded by the sphere \( x^2 + y^2 + z^2 = 4 \) and the coordinate planes. Evaluate \( \iiint_{G} xyz \, dV \) using
   (a) rectangular coordinates;
   (b) cylindrical coordinates;
   (c) spherical coordinates.
ASSIGNMENT 10

1. Let $\mathbf{F} = (3x+2y)\mathbf{i} + (2x-y)\mathbf{j}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $C$ is
   (a) the line segment from $(0,0)$ to $(1,1)$ (b) the curve $y = x^2$ from $(0,0)$ to $(1,1)$
   (c) the curve $y = \sin(\pi x/2)$ from $(0,0)$ to $(1,1)$

2. Find the work done by a force $\mathbf{F}(x,y,z) = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$ acting on a particle that moves along the curve $\mathbf{r}(t) = ti + t^2j + t^3k, 0 \leq t \leq 1$.

3. Show that $\int_{(0,1)}^{(\pi,-1)} y \sin x \, dx - \cos x \, dy$ is independent of the path, and evaluate the integral by
   (a) using the fundamental theorem of line integrals (Theorem 18.2.1 in Anton);
   (b) integrating along the line segment from $(0,1)$ to $(\pi,-1)$.

4. Show that the integral $\int_{(0,0)}^{(1,\pi/2)} e^x \sin y \, dx + e^x \cos y \, dy$ is independent of the path, and find its value by any method.

5. Use Green’s Theorem to calculate $\int_C (e^x + y^2)dx + (e^y + x^2)dy$, where $C$ is the boundary of the region between $y = x^2$ and $y = x$ oriented counterclockwise.

6. Calculate $\int_C -\frac{y}{x^2 + y^2}dx + \frac{x}{x^2 + y^2}dy$ where $C$ is the circle $x^2 + y^2 = 1$ oriented counterclockwise. State whether Green’s Theorem can be used and why.
ASSIGNMENT 11

1. Evaluate the surface integral $\iint_{\sigma} xyz \, dS$ where $\sigma$ is the portion of the plane $x+y+z=1$ lying in the first octant.

2. Evaluate the surface integral $\iint_{\sigma} (x^2 + y^2) \, dS$, where $\sigma$ is the portion of the sphere $x^2 + y^2 + z^2 = 4$ above the plane $z = 1$.

3. Use any appropriate formula to calculate the indicated unit normal.
   (a) The unit normal to the surface $y^2 = x$ at $(1,1,2)$ that points toward the $xz$-plane.
   (b) The unit normal to the surface $y = z^2 - x^2$ at $(1,3,2)$ that points toward the $yz$-plane.

4. Evaluate $\iint_{\sigma} \mathbf{F} \cdot \mathbf{n} \, dS$ where
   
   $\mathbf{F}(x, y, z) = (x + y)\mathbf{i} + (y + z)\mathbf{j} + (z + x)\mathbf{k}$ and $\sigma$ is the portion of the plane $x+y+z=1$ in the first octant, oriented by unit normals with positive components.

5. Use the Divergence Theorem to evaluate $\iint_{\sigma} \mathbf{F} \cdot \mathbf{n} \, dS$, where $\mathbf{n}$ is the outer unit normal to $\sigma$.
   (a) $\mathbf{F}(x, y, z) = 2xi + 2yj + 2zk$, $\sigma$ is the sphere $x^2 + y^2 + z^2 = 9$.
   (b) $\mathbf{F}(x, y, z) = z^3\mathbf{i} - x^3\mathbf{j} + y^3\mathbf{k}$, $\sigma$ is the sphere $x^2 + y^2 + z^2 = a^2$. 
ASSIGNMENT 12

1. Use Stokes’ Theorem to evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where

\[
\mathbf{F}(x, y, z) = -3y^2 \mathbf{i} + 4z \mathbf{j} + 6x \mathbf{k},
\]

\( C \) is the triangle in the plane \( z = (1/2)y \) with vertices (2, 0, 0), (0, 2, 1) and (0, 0, 0), with a counterclockwise orientation looking down the positive \( z \)-axis.

2. Verify Stokes’s Theorem by direct calculations of \( \int_C \mathbf{F} \cdot d\mathbf{r} \) and \( \int \int_{\sigma} (\text{curl} \mathbf{F}) \cdot \mathbf{n} \, dS \) for

\[
\mathbf{F}(x, y, z) = x^2 \mathbf{i} + y^2 \mathbf{j} + z^2 \mathbf{k},
\]

where \( \sigma \) is the portion of the cone \( z = (x^2 + y^2)^{1/2} \) below the plane \( z = 1 \).

3. Show that if the components of \( \mathbf{F} \) and their first- and second-order partial derivatives are continuous, then \( \text{div}(\text{curl}\mathbf{F}) = 0 \).

4. Find the net volume of the fluid that crosses the surface \( \sigma \) in the direction of the orientation in one unit of time where the flow field is given by \( \mathbf{F}(x, y, z) = x^2 \mathbf{i} + yx \mathbf{j} + zx \mathbf{k} \), \( \sigma \) is the portion of the plane \( 6x + 3y + 2z = 6 \) in the first octant oriented by upward unit normals.