Sample solutions for tutorial 3

**Question 1.** Solve the following equations for the real number $x$.

(a) $3e^{2x} = 9$

$e^{2x} = \frac{9}{3} = 3$, $2x = \ln 3$, $x = \frac{1}{2} \ln 3$.

(b) $\frac{1}{2} \ln x - 3 = 0$

$\frac{1}{2} \ln x = 3$, $\ln x = 3 \times 2 = 6$, $x = e^6$.

**Question 2.** Given that $\ln 3$ is approximately 1.099 and $\ln 4$ approximately 1.386, calculate an approximation for

(a) $\ln 18$

$\ln 18 = \ln (2 \times 9) = \ln (4^{\frac{1}{2}} \times 3^2) = \ln (4^{\frac{1}{2}}) + \ln 3^2 = \frac{1}{2} \ln 4 + 2 \ln 3$

$\approx \frac{1}{2} \times 1.386 + 2 \times 1.099 = 2.891$.

(b) $\ln \frac{4}{9}$

$\ln \frac{4}{9} = \ln 4 - \ln 9 = \ln 4 - \ln 3^2 = \ln 4 - 2 \ln 3 \approx 1.386 - 2 \times 1.099 = -0.812$.

**Question 3.** Sketch the graphs of the following functions for $-\pi \leq x \leq 2\pi$.

See the sketch on the next page.

Sketch the graph of the function $\sin x$

(a) $f_1(x) = 2\sin x$. Stretch the graph of the function $\sin x$ by the factor of 2 along the y-axis.

(b) $f_2(x) = -2\sin x$. Make a reflexion of the graph $f_1(x) = 2\sin x$ in the x-axis.

(c) $f_3(x) = 2\sin(4x)$. Compress the graph of the function $f_1(x) = 2\sin x$ by the factor 4 along the x-axis.

(d) $f_4(x) = 2\sin(x + \frac{\pi}{4})$. Shift the graph of the function $f_1(x) = 2\sin x$ to the left by $\frac{\pi}{4}$. 
(e) $f_5(x) = 2\sin x + 1$. Shift the graph of the function $f_1(x) = 2\sin x$ upwards by 1 unit.
Question 4. Sketch on the one diagram the graphs of the functions given by

(a) $g_1(x) = \ln x, \ x > 0$ - see the sketch on the extra page

(b) $g_2(x) = -\ln x, \ x > 0$ - make a reflexion of the graph $\ln x$ in the x-axis.

(c) $g_3(x) = \ln(x - 1), \ x > 1$ - shift the graph of the function $\ln x$ to the right by 1 unit.
Question 5. For each function find the inverse. Sketch both the function and its inverse in one diagram.

(a) \( y = 18x \). In order to find the inverse function we first switch the variables \( x \) and \( y \): \( x = 18y \). Then we solve for \( y \): \( y = \frac{1}{18}x \).

(b) \( y = \ln(x - 4) \), for \( x > 4 \). Switch the variables \( x \) and \( y \): \( x = \ln(y - 4) \), which is defined only if \( y - 4 > 0 \), i.e. \( y > 4 \). Solve for \( y \): \( y - 4 = e^x \), \( y = e^x + 4 \).

Note that in each case the inverse of the function is the reflexion of the function itself about the straight line \( y = x \).