MATH110 — Assignment 9

Solutions

Question 1.

The first step is to sketch the graph of $y = x^3 - x^2$. The function y has zeros at $x = 0$ and $x = 1$ and is negative between these values (see graph)

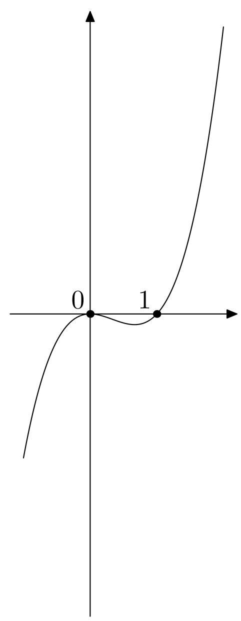


Figure 1: Graph of $y = x^3 - x^2$

Therefore the area between the graph and the x-axis is

$$-\int_0^1 (x^3 - x^2)dx = -\left[\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_0^1 = -\left[\frac{1}{4} - \frac{1}{3}\right] + 0 = \frac{1}{12}.$$

Question 2.

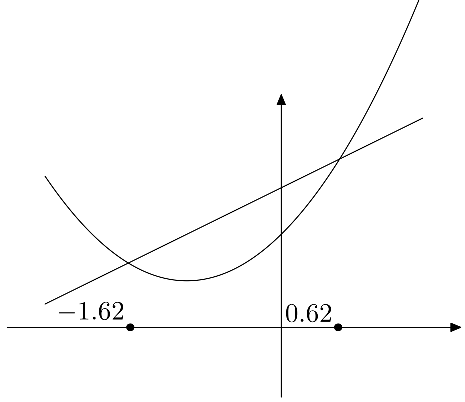
Again the first step is to sketch the graphs and to calculate their points of intersection. Solving

$$x + 3 = x^2 + 2x + 2 \qquad \text{gives} \qquad x^2 + x - 1 = 0$$

and the quadratic formulas shows that the functions intersect at $x = \frac{1}{2}(-1 \pm \sqrt{5})$. For x between these numbers we have $x + 3 > x^2 + 2x + 2$ (see graph).

Thus the required area is

$$\int_{\frac{1}{2}(-1-\sqrt{5})}^{\frac{1}{2}(-1+\sqrt{5})} (x + 3 - x^2 - 2x - 2)dx = \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]_{\frac{1}{2}(-1-\sqrt{5})}^{\frac{1}{2}(-1+\sqrt{5})}.$$



Question 3.

The differential equation we need to solve is

$$\frac{dy}{dt} = 0.1y$$

where $y =$ height of the plant and t is the time in days. The solution is

$$y(t) = y_0 e^{0.1t}, \qquad y_0 = 0.1 \text{ meters.}$$

At 8pm Friday $t = 4.5$ and

$$y(t) = 0.1e^{0.1\times 4.5} = 0.157 \text{ meters.}$$

Let T be the time to reach 2 meters. Then

$$y(T) = 2 = 0.1e^{0.1T}.$$

Taking logarithms gives

$$\ln 2 = \ln 0.1 + 0.1T$$

and so

$$T = \frac{\ln 2 - \ln 0.1}{0.1} = 29.95 \text{ days.}$$

Question 4.

The differential equation is

$$\frac{dy}{dt} = -ky \qquad k = 9.2 \times 10^{-6}$$

where y is the quantity of thorium and t is the time in years. Using the formula for the half-life

$$T = \frac{\ln 2}{k} = 7.53 \times 10^4 \text{ years.}$$

The solution of the differential equation is

$$y(t) = y_0 e^{-kt}$$

where y_0 is the original quantity of thorium. After 200 years

$$y(200) = y_0 e^{-18.4\times 10^{-4}} = 0.9982y_0$$

so the fraction of thorium which has decayed is

$$1 - 0.9982 = 0.0018.$$