MATH110 — Assignment 7

Solutions

Question 1.

Let the enclosure have sides of width x and length y. Then the area of the enclosure is $3610 = xy$ and so

$$y = \frac{3610}{x}.$$

Thus the total cost of fencing is

$$C = 20x + 10y + 62y = 20x + \frac{259920}{x}.$$

To find the minimum of $C(x)$ we differentiate with respect to x

$$C'(x) = 20 - \frac{259920}{x^2},$$

set the derivative to zero

$$20 - \frac{259920}{x^2} = 0,$$

and solve for x

$$x^2 = \frac{259920}{20} \qquad \Longrightarrow \qquad x = 114m.$$

The length of the enclosure is

$$y = \frac{3610}{114} = 31.7m$$

and the total cost of fencing

$$C(114) = 20(114) + \frac{259920}{114} = \$4560.$$

Question 2.

(a) The derivative is

$$y' = 15x^2 - 2x - 1.$$

Solving $y' = 0$ via the quadratic formula, we see that

$$x = \frac{2 \pm \sqrt{64}}{30} = \frac{1}{3}, \frac{-1}{5}.$$

The second derivative is

$$y'' = 30x - 2$$

so that $y''(1/3) = 8$ implies the function has a local minimum at $x = 1/3$, while $y''(-1/5) = -8$ implies the function has a local maximum at $x = -1/5$.

(b) The derivative is

$$y' = 4x^3 - 6x^2 = 2x^2(2x - 3).$$

Solving $y' = 0$ gives

$$x = 0 \qquad \text{and} \qquad x = \frac{3}{2}.$$

The second derivative is

$$y'' = 12x^2 - 12x = 12x(x-1).$$

For $x = 0$, we find $y''(0) = 0$ and the second derivative test gives us no information. Therefore we need to apply the first derivative test, that is examine the sign of $y' = 2x^2(2x-3)$ near $x = 0$. Now for x small and negative

$$y' = 2x^2(2x-3) < 0$$

since one factor is negative. For x small and positive

$$y' = 2x^2(2x-3) < 0,$$

since again the first factor is positive and the second factor is negative. Thus the function has an inflection point at $x = 0$. On the other hand, at $x = 3/2$

$$y'' = (12)(3/2)(3/2-1) > 0$$

and the second derivative test shows that this point is a local minimum.
(c) The product rule shows that the derivative is

$$y' = x^2e^x + 2xe^x = x(x+2)e^x.$$

Solving for $y' = 0$ gives

$$x = 0 \qquad \text{and} \qquad x = -2.$$

The second derivative is

$$y'' = x^2e^x + 4xe^x + 2e^x.$$

Now $y''(0) = 2 > 0$ and so $x = 0$ is a local minimum.

$$y''(-2) = -2e^{-2} < 0$$

so $x = -2$ is a local maximum.