MATH110 — Assignment 6

Solutions

Question 1.

(a)

$$\frac{dy}{dx} = 20x^3 + 3 + 3x^{-4}.$$

(b) Use the chain rule:

$$y = u^{-\frac{1}{3}}, \qquad u = x + 1$$

Thus

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} \\ &= -\frac{1}{3}u^{-\frac{4}{2}}(1) \\ &= -\frac{1}{3}(x+1)^{-\frac{4}{3}} \end{aligned}$$

(c) Another chain rule:

$$y = e^u, \qquad u = \sqrt{x+1}$$

giving

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2\sqrt{x+1}}e^{\sqrt{x+1}}$$

(d) Use the product rule:

$$y = uv, \qquad u = e^{3x}, \qquad v = \ln x$$

Thus

$$\begin{aligned} \frac{dy}{dx} &= v\frac{du}{dx} + u\frac{dv}{dx} \\ &= \ln x 3e^{3x} + e^{3x}\frac{1}{x} \\ &= (3\ln x + 1/x)e^{3x}. \end{aligned}$$

(e) The chain rule:

$$\frac{dy}{dx} = \frac{1}{x^2 + x^{-2}}(2x - 2x^{-3}) = \frac{2(x^2-1)}{x}.$$

(f) The quotient rule:

$$\frac{dy}{dx} = \frac{-1 - 2\sin x}{(1-\cos x)^2}.$$

Question 2.

$$\frac{dy}{dx} = -e^x \sin x + e^x \cos x$$

so

$$\frac{d^2y}{dx^2} = -e^x \sin x - e^x \cos x + e^x \cos x - e^x \sin x = -2e^x \sin x.$$

Question 3.

From question 5(a)

$$\frac{d^2y}{dx^2} = 60x^2 - 12x^{-5}.$$

Thus

$$\frac{d^3y}{dx^3} = 120x + 60x^{-6}$$

and

$$\frac{d^4y}{dx^4} = 120 - 360x^{-7}$$

Question 4.

The derivative is

$$\frac{dy}{dx} = 6x^2 + 3.$$

Thus when $x = 1$, $y = 4$ and $\frac{dy}{dx} = 9$ and we need to find the straight line through the point $(1, 9)$ with slope 9. The line is

$$y = 9x - 5.$$