

# MATH110 — Assignment 2

## Solutions

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Question 1.

(a)

$$\mathbf{A}^2 = \begin{bmatrix} 1 & 2 & 2 \\ -4 & 13 & 10 \\ -1 & 3 & 4 \end{bmatrix},$$

$$\mathbf{A}^3 = \begin{bmatrix} -1 & 8 & 10 \\ -17 & 49 & 44 \\ -4 & 13 & 10 \end{bmatrix},$$

(b) Show by direct multiplication that  $\mathbf{A} \times \mathbf{A}^{-1} = \mathbf{I}$ .

(c)

$$\mathbf{X} = \mathbf{A}^{-1} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix}.$$

Question 2.

(a) The determinant of  $\mathbf{C} = 1 \times 2 - 2 \times 3 = -4$ .

(b)

$$\mathbf{C}^{-1} = \begin{bmatrix} -1/4 & 1/2 \\ 3/4 & -1/2 \end{bmatrix}.$$

Question 3.

(a) 0, 0.1, 0.5 .

(b) 0.8, 0.8 .

(c)

$$\mathbf{L}^2 = \begin{bmatrix} 0.05 & 0.8 & 0.4 & 0 \\ 0 & 0.05 & 0.5 & 0.25 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.64 & 0 & 0 \end{bmatrix},$$

(d) Population vector after one year is

$$\begin{bmatrix} 1520 \\ 500 \\ 960 \\ 880 \end{bmatrix}.$$

(e)

$$\mathbf{P}_2 = \mathbf{L}^2 \mathbf{P}_0 = \begin{bmatrix} 1450 \\ 760 \\ 400 \\ 768 \end{bmatrix},$$

(f) Check by direct multiplication that  $\mathbf{L} \times \mathbf{L}^{-1} = \mathbf{I}$ .

(g)

$$\mathbf{P}_{-1} = \mathbf{L}^{-1}\mathbf{P}_0 = \begin{bmatrix} 2400 \\ 1375 \\ 750 \\ 225 \end{bmatrix}.$$