Chapter 1

1. All vectors have dimension 5.
2. \( Z_1 = 0, Z_4 = -2, Z_5 = 1 \).
3. \[
\begin{bmatrix}
0 \\
5 \\
-2 \\
11 \\
-4
\end{bmatrix}
\]
4. \[
\begin{bmatrix}
0 \\
9 \\
0 \\
-6 \\
3
\end{bmatrix}
\]
5. \[
\begin{bmatrix}
-1/2 \\
3/2 \\
-5/2 \\
7/2 \\
-9/2
\end{bmatrix}
\]
6. 2.
7. -14.
8. \( \sum_{i=1}^{5} Y_i^2 = 165, (\sum_{i=1}^{5} Y_i)^2 = 25 \).
9. \( \langle X, X \rangle = \sum_{i=1}^{n} X_i^2 \). Since each term \( X_i^2 \geq 0 \) the sum is \( \geq 0 \).

Chapter 2

1. All are \( 4 \times 3 \) matrices.
2. \( A_{12} = 2, A_{23} = -3 \) and \( A_{41} = 3 \).
3. \[
A + B = \begin{bmatrix}
1 & 2 & 3 \\
1 & 0 & -6 \\
2 & 2 & -2 \\
-1 & -3 & -2
\end{bmatrix}
\]
4. \[
2C = \begin{bmatrix}
-6 & -4 & 4 \\
2 & 6 & 10 \\
-4 & -2 & 0 \\
-2 & -2 & -4
\end{bmatrix}
\]
5. 
\[ A + B - 2C = \begin{bmatrix} 7 & 6 & -1 \\ -1 & -6 & -16 \\ 6 & 4 & -2 \\ 1 & -1 & 2 \end{bmatrix} \]

6. 
\[ B = A - C = \begin{bmatrix} -1 & -3 & 1 & 6 \\ -4 & -2 & 10 & 5 \end{bmatrix} \]

7. \(B - A = \) the increase in mosquito populations over the year.
\[ B - A = \begin{bmatrix} 1262 & 709 & 70 & 26 \\ 1170 & 653 & 59 & 57 \\ 211 & 337 & 26 & 20 \end{bmatrix} \]

8. 
\[ \frac{1}{2}(B - A) = \begin{bmatrix} 631.5 & 354.5 & 35.0 & 13.0 \\ 585.0 & 326.5 & 29.5 & 28.5 \\ 105.5 & 168.5 & 13.0 & 10.0 \end{bmatrix} \]

9. 
\[ \text{Total} = A + B = \begin{bmatrix} 153 & 82 & 33 \\ 43 & 68 & 55 \\ 56 & 60 & 37 \end{bmatrix} \]

10. 
\[ \text{Change} = A - B = \begin{bmatrix} -43 & 4 & -11 \\ -5 & 38 & 25 \\ -28 & 16 & 3 \end{bmatrix} \]

11. 
\[ \text{Total} = \frac{1}{2}(A + B) = \begin{bmatrix} 76.5 & 41.0 & 16.5 \\ 21.5 & 34.0 & 27.5 \\ 28.0 & 30.0 & 18.5 \end{bmatrix} \]

12. 
\[ A = \begin{bmatrix} 300 & 217 & 64 \\ 271 & 201 & 83 \end{bmatrix} \]

13. 
\[ B = \begin{bmatrix} 284 & 196 & 58 \\ 288 & 202 & 69 \end{bmatrix} \]
\[ C = \begin{bmatrix} 184 & 112 & 49 \\ 318 & 213 & 73 \end{bmatrix} \]

14. (i) 
\[ A + B = \begin{bmatrix} 584 & 413 & 122 \\ 559 & 403 & 152 \end{bmatrix} \]

(ii) 
\[ C - B = \begin{bmatrix} -100 & -84 & -9 \\ 30 & 11 & 4 \end{bmatrix} \]
15. (i) $\mathbf{A} + \mathbf{B} =$ Total production on the first two days.
   (ii) $\mathbf{C} - \mathbf{B} =$ Increase in production from day 2 to day 3.
   (iii) $\frac{1}{2}(\mathbf{B} + \mathbf{C}) =$ Average production on the second and third days.
   (iv) $\frac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C}) =$ Average production over the three days.
   (v) $\frac{2}{3}\mathbf{A} - \frac{1}{3}\mathbf{B} - \frac{1}{3}\mathbf{C} =$ Excess of production on day 1 over the average production for the three days.

16. $\mathbf{A}$ is $2 \times 3$, $\mathbf{B}$ is $2 \times 2$ and $\mathbf{C}$ is $3 \times 1$.

17. The products $\mathbf{A} \times \mathbf{C}$, $\mathbf{B} \times \mathbf{A}$ and $\mathbf{B} \times \mathbf{B}$ are defined.

18.

$$
\mathbf{A} \times \mathbf{C} = \begin{bmatrix} 8 \\ -7 \end{bmatrix}
$$

$$
\mathbf{B} \times \mathbf{A} = \begin{bmatrix} -2 & 7 & 2 \\ 2 & -10 & -3 \end{bmatrix}
$$

$$
\mathbf{B} \times \mathbf{B} = \begin{bmatrix} 3 & -8 \\ -4 & 11 \end{bmatrix}
$$

19.

$$
\mathbf{AC} = \begin{bmatrix} 2816 \\ 954 \\ 1430 \end{bmatrix}
$$

This is the total amount of vegetation eaten in the three areas in 1958.

20.

$$(\mathbf{A} + \mathbf{B})\mathbf{C} = \begin{bmatrix} 4687.5 \\ 2860.5 \\ 2693.0 \end{bmatrix}
$$

21.

$$
\mathbf{AB} = \begin{bmatrix} 2200 & 3000 \\ 2750 & 3700 \end{bmatrix}
$$

22. This is the cost of removing pollutants from products at the plants. The rows correspond to the products, the columns the plants.

23.

$$
\mathbf{L} = \begin{bmatrix} 0 & 30 & 30 & 40 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}
$$
24.

\[ P_1 = LP = \begin{bmatrix} 2000 \\ 0 \\ 10 \\ 10 \end{bmatrix} \]

25.

\[ P_2 = LP_1 = \begin{bmatrix} 700 \\ 1800 \\ 0 \\ 5 \end{bmatrix} \]

\[ P_3 = LP_2 = \begin{bmatrix} 54200 \\ 630 \\ 900 \\ 0 \end{bmatrix} \]

\[ P_4 = LP_3 = \begin{bmatrix} 45900 \\ 48780 \\ 315 \\ 450 \end{bmatrix} \]

26.

\[ A^2 = \begin{bmatrix} 1 & -6 \\ 0 & 4 \end{bmatrix} \]

27.

\[ A^3 = \begin{bmatrix} 1 & -14 \\ 0 & 8 \end{bmatrix} \]

\[ A^4 = \begin{bmatrix} 1 & -30 \\ 0 & 16 \end{bmatrix} \]

28.

\[ A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{bmatrix} \]

29.

Both = \[ \begin{bmatrix} 1 & 1 \frac{1}{3} \\ 0 & \frac{1}{4} \end{bmatrix} \]

30. Just check that \( LL^{-1} = I \).

31.

\[ P_{-1} = L^{-1}P = \begin{bmatrix} 222.22 \\ 0 \\ 10 \\ 10 \end{bmatrix} \]

32.

\[ P_{-2} = L^{-1}P_{-1} = \begin{bmatrix} 0 \\ 20 \\ 20 \\ -24.44 \end{bmatrix} \]
\[ \mathbf{P}_{-3} = \mathbf{L}^{-1}\mathbf{P}_{-2} = \begin{bmatrix} 22.22 \\ 40 \\ -48.89 \\ 6.67 \end{bmatrix} \]

Chapter 3

1.

2. (i) \( y = \frac{1}{5}x + 7 \).
   (ii) \( y = 8x \).

3. \( y = -2x + 11 \).

4. The approximate equations are \( y = 6x - 30 \) and \( y = -3.3x + 10 \).

5. \( y = -1.5x + 3 \).

6. \( 0.05x + 0.08y = 1 \).
7. (i) $y = 0.6x - 10$.
(ii) $b =$ rate of change of respiratory rate with partial pressure of CO$_2$.
(iii) 17.
(iv) -4. A negative respiratory rate is impossible.
(v) No. It may give a good description under normal conditions, but at low levels of CO$_2$ it is obviously wrong as the answer to (iv) shows.

8.

![Graph of quadratic equations]

9.

![Graph of quadratic equations]

10. Yield = $x(500 - 5x) = -5x^2 + 500x$. The maximum yield occurs on the axis of symmetry of the quadratic, i.e. at $x = 50$, giving a maximum yield of 12500 lb per acre.

11. (i) 64, (ii) 8, (iii) 1, (iv) $8^{2/3} = \sqrt[3]{8^2} = 4$, (v) 1/8, (vi) 1/64, (vii) $8^{-1/3} = 1/\sqrt[3]{8} = 1/2$. 
12. \[ y = x^{-1}, \quad y = x^4, \quad y = x^{1/3} \]

13. \[ y = 3^x, \quad y = 2^x, \quad y = 3^{-x} \]

14. (i) \( t = 0 \) gives initial amount = \( A \).
15. (i) 

(ii) $y \rightarrow 100$.

(iii) $y \rightarrow 0$.

Very large amounts of fertilizer would probably kill the crop.

16.

\[
\begin{align*}
\ln 6 &= \ln(2.3) = \ln 2 + \ln 3 = 1.792 \\
\ln 64 &= \ln(2^6) = 6 \ln 2 = 4.158 \\
\ln \left(\frac{1}{128}\right) &= \ln(2^{-7}) = -7 \ln 2 = -4.851 \\
\ln \left(\frac{8}{9}\right) &= \ln \left(\frac{2^3}{3^2}\right) = 3 \ln 2 - 2 \ln 3 = -0.119
\end{align*}
\]

17. (i) (a) 3.51 (b) 9.68

(ii) $10^{-3}$

18. (i) $10^3$

(ii) $8.9 - \log 4 = 8.30$

(iii) $10^{(8.9-0.23)} = 470$

19. height = $7 \tan 23 = 2971m$

20. (i) $1/2$, (ii) 1, (iii) $-1/2$, (iv) -1, (v) -1, (vi) 0, (vii) -1, (viii) 1
21. (i) 

(ii)
22. (i)

$$\begin{array}{c}
\text{(ii) (a) } 0.1047, \text{ (b) } 0.2035, \text{ (c) } 51.28 \\
\text{(iii) } 512.8 \\
\text{(iv) No. There is a limit to the number of trees a given area of land can support, so this would indicate that the relationship is incorrect for large } S, \text{ although in practical terms the yield is so low it doesn’t matter. For small } S \text{ the relationship predicts arbitrarily large yields which is inconsistent with there being an upper limit to the yield per tree.}
\end{array}$$
Chapter 4

1. \( x = 1 \)
2. \( x = \frac{-3 + \sqrt{17}}{2} \)
3. \( x = 1.5^{2/3} = 1.3104 \)
4. \( x = \frac{\ln 2}{2} = 0.3466 \)
5. \( x = \frac{7}{8} \) (and other answers)
6. Eliminating \( y \) gives \( x = -5\frac{1}{2} \) and then \( y = -10\frac{1}{2} \).
7. Eliminating \( y \) gives \( x^2 - 4x + 4 = 0 \)

and solving gives \( x = 2, y = -2 \).

Chapter 5

1. The average rate of change is \( \frac{p(7) - p(2)}{5} = 0.4 \)
2. The average rate of change is

\[ \frac{f(1 + \Delta x) - f(1)}{\Delta x} = \frac{\Delta x + 2\Delta x^2}{\Delta x} = 1 + 2\Delta x \]

Then

\[ f'(1) = \lim_{\Delta x \to 0} (1 + 2\Delta x) = 1 \]
3. The average rate of change is

\[ \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{-3\Delta x + 4x\Delta x + 2\Delta x^2}{\Delta x} = -3 + 4x + 2\Delta x \]

Then

\[ f'(x) = \lim_{\Delta x \to 0} (-3 + 4x + 2\Delta x) = -3 + 4x \]
4. The definition of the derivative gives \( f'(3) = -14 \) and the equation of the tangent is \( y = -14x + 30 \).
5. (a) \( y' = 60x^5 \)
   (b) \( y' = 24x^7 + 30x^4 - 8x \)
   (c) \( y' = 7.5t^{6.5} \)
   (d) \( y = x^2 + 2 - 3x^{-3} \), so \( y' = 2x + 9x^{-4} \)
   (e) \( u' = -15t^{-4} + 8t \)
   (f) \( y = x^{7/6} \), so \( y' = \frac{7}{6}x^{1/6} \)
   (g) Expanding gives \( v = 2x^3 + 2x^2 - 12x \) and \( v' = 6x^2 + 4x - 12 \).
6. \( y = 8x - 1 \).
7. (a) Product rule. \( y' = (2x - 4)(x^3 + 4) + (x^2 - 4x + 3)3x^2 \)
   (b) Chain rule. \( y' = \frac{7}{3}(4x^3 + 9x^2)(x^4 + 3x^3 + 1)^{-2/3} \)
   (c) Chain rule. \( y' = 3x(x^2 + 7)^{-1/2} \)
(d) Quotient rule. \( y' = \frac{2x(3x^3 + 4) - 9x^2(x^3 - 3)}{(3x^3 + 4)^2} \)
(e) \( y = 2x^{3/2} + x^{-1/2}, \quad y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-3/2} \)
8. \( u = \frac{t^2 + 4}{2t^2 - 3t}, \quad u' = \frac{2(t^2 - 3t) - (4t - 3)(t^2 + 4)}{(2t^2 - 3t)^2} \)
9. (i) \( m_0 \) is the mass at \( t = 0 \).
(ii) \( \frac{dm}{dt} = (\frac{1}{2}t + m_0^{1/3})^2 \)
(iii) \( m_0^{2/3}, \quad (\frac{3}{8} + m_0^{1/3})^2 \)
(iv) No. Since \( \frac{dm}{dt} \) is a square it is never negative, the mass cannot decrease.
10. \( \frac{dw}{dx} = 15x^4 + 21x^2 - 8x \quad \frac{d^2y}{dx^2} = 60x^3 + 42x - 8, \quad \frac{d^3y}{dx^3} = 0. \)
11. (a) \((t + 1)^{-4} \)
(b) \((t^2 + 1)^{-3/2} \)
12. (a) The velocity is \( s' = 3 - 2t \) which is zero at \( t = \frac{3}{2} \).
(b) The acceleration is \( s'' = 2 \) (the same for all \( t \)).
13. (a) Chain rule. \( y' = 2xe^{x^2} \)
(b) Product rule. \( y = x^{-1}e^x, \quad y' = -x^{-2}e^x + x^{-1}e^x \)
(c) Chain rule. \( y = e^{\sqrt{x} - x}, \quad y' = (\frac{1}{2}x^{-1/2} - 1)e^{\sqrt{x} - x} \)
(d) \( y = \ln(x^4) = 4 \ln x, \quad y' = 4/x \)
(e) Chain rule. \( y' = \frac{5}{2}(\ln x)^4 \)
(f) Product rule. \( y' = \frac{1}{2}e^x + \ln xe^x \)
(g) Product and chain rules. \( y' = 2x \ln(x^2 + x) + \frac{x^2(2x + 1)}{x^2 + x} \)
(h) \( y = \ln e^x + \ln(x^2 - 1) = \ln x^3 = x + \frac{1}{2} \ln(x - 1) - 3 \ln x, \quad y' = 1 + \frac{1}{2} \frac{1}{x - 1} - \frac{3}{x} \)
14. \( y' = 3kekpe^{-kt}(1 - e^{-kt})^2 \)
15. (a) \( y' = 3 \cos(3x) \)
(b) \( y' = -x^{-2} \cos x - x^{-1} \sin x \)
(c) \( y' = x^{-1/2} \sin(\sqrt{x}) \cos(\sqrt{x}) \)
(d) \( y' = 15 \frac{2}{x}x^2(x^3 + 1)^{-1/2} \cos x - 5\sqrt{x^3 + 1} \sin x \)
(e) \( y' = -2 \sin(2x)e^{\cos(2x)} \)
(f) \( y' = -\sin \frac{\cos x}{1 + \cos x} \)
(g) \( y' = -\frac{\cos x}{\sin x} \) (using \( \sin^2 x + \cos^2 x = 1 \))
16. (a) \( y'' = 2 \cos x - x \sin x \)
(b) \( y'' = 2(\cos^2 x - \sin^2 x) \)
17. The rate of growth is
\[ w' = e^t(2 - \cos t + \sin t) \]
\[ w'(0) = e, \quad w'(\frac{\pi}{2}) = 3e^{\pi/2}, \quad w'(2\pi) = e^{2\pi}. \]
\( w \) can never decrease, since \( w' \) is never negative. To see this note that \( \sin x \) and \( \cos x \) are never greater than 1 in magnitude, so the term in brackets in the formula for \( w' \) is never negative.

Chapter 6

1. (a) \( x = 1, y = -22, \) local minimum; \( x = -2, y = 5, \) local maximum.
   (b) \( x = \frac{1}{2}, y = \frac{1}{2}e^{-1}, \) local maximum.
2. The maximum at \( t = 1 \) is \( \frac{1}{2\pi}. \)
3. The only stationary point is a local minimum at $t = 4$. This indicates that the number of bacteria at first decrease, but then increase again after 4 hours. Further application of the drug will be necessary to permanently decrease the bacteria count.

4. $R = \frac{4}{7}R_0$

5. The maximum of $A(\frac{5}{6})^{2.5}e^{-2.5}$ occurs at $t = \frac{5}{6}$.

Chapter 7

1. (a) $\int x^7 dx = \frac{1}{8}x^8$
   (b) $\int x^{1/2} dx = \frac{2}{3}x^{3/2}$
   (c) $\int (x^{-1/2} + 3x^{-2}dx = 2x^{1/2} - 3x^{-1}$
   (d) $\int 2e^{-x/2}dx = -4e^{-x/2}$
   (e) $\int (3\sin(x/2) + 2\cos(2x))dx = -6\cos(x/2) + \sin(2x)$
   (d) $\int \frac{1}{8}x^8 + \frac{25}{7}x^2 - 7\ln(x)$

2. All of these require that that functions to be integrated are expanded before integration.
   (a) $\int (\frac{4}{3}t + \frac{4}{3} + \frac{1}{3}t^{-1})dx = \frac{2}{3}t^2 + \frac{4}{3}t + \frac{1}{3}\ln(t)$
   (b) $\int (\frac{2}{3}y^2 + \frac{7}{3}y - 2 + 3y^{-1})dy = \frac{2}{9}y^3 + \frac{7}{6}y^2 - 2y + 3\ln(y)$
   (c) $\int (2x^2 + x - 1)dx = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x$
   (b) $\int (t^{5/2} - 2t^{3/2} + t^{1/2})dt = \frac{2}{7}t^{7/2} - \frac{4}{5}t^{5/2} + \frac{2}{3}t^{3/2}$

3. (1a) $\leftrightarrow$ (2d), (1b) $\leftrightarrow$ (2a), (1c) $\leftrightarrow$ (2b), (1d) $\leftrightarrow$ (2c).

4. $\int_0^4 32tdt = 256 ft$.

5. The velocity is

$$\frac{dh}{dt} = \int \frac{d^2h}{dt^2}dt = -32t + c$$

The initial condition gives $c = 1000$. Now

$$h(t) = \int \frac{dh}{dt}dt = \int (-32t + 1000)dt = -16t^2 + 1000t$$

The maximum altitude occurs when $\frac{dh}{dt} = 0$, i.e $t = \frac{125}{4}$ and $h(t) = 15625$ ft. The bullet returns to earth when $h(t) = 0$, i.e. at $t = \frac{125}{2}$ secs.

6. The rate of blood flow is $v = 0.07(150 - 3t)$. Therefore the answer is

$$\int_0^{20} 0.07(150 - 3t)dt = 168 \text{ litres}$$

7. In each of these the function is positive on the interval of integration.
   (a) Area = $\int_{\pi/2}^{\pi} \cos xdx = [\sin x]_{\pi/2}^{\pi} = 1$
(b) Area = \( \int_{-2}^{2} (e^{2x} + 7)dx = \left[ \frac{1}{2}e^{2x} + 7x \right]_{-2}^{2} = \frac{1}{2}(e^{4} - e^{-4}) + 28 \)

(c) Area = \( \int_{1}^{8} (3x^{1/2} + x^{-1})dx = [2x^{3/2} + \ln x]_{1}^{8} = 32\sqrt{2} + \ln 8 - 2 \)

8. (a) Integral = -9, Area = 9.
(b) Integral = 0, Area = 8.
(c) Integral = 2, Area = 6.
(d) Integral = \( \frac{16}{3} \), Area = \( \frac{17}{3} \).

9. (a) Area = \( \int_{1}^{2} (5x - 2x^{2})dx = \frac{17}{6} \)
(b) Area = \( \int_{0}^{1} (e^{x} - x^{2})dx = e - \frac{4}{3} \)

10.
For drug A: \( F_A(t) = \int 10e^{-t} dt = -10e^{-t} \)
For drug B: \( F_B(t) = \int (10e^{-t/2} - 8e^{-2t}) = -20e^{-t/2} + 4e^{-2t} \)

(i) \( F_A(1) - F_A(0) = 10(1 - e^{-1}) = 6.32, F_B(1) - F_B(0) = 16 - 20e^{-1/2} + 4e^{-2} = 4.41 \)
Drug A is more effective.

(ii) \( F_A(5) - F_A(0) = 10(1 - e^{-5}) = 9.93, F_B(5) - F_B(0) = 16 - 20e^{-5/2} + 4e^{-10} = 14.36 \)
Drug B is more effective.

(iii) The averages from \( t = 0 \) to \( t = 5 \) are: drug A 1.99, drug B 2.87

Chapter 8

1. The differential equation is
\[ \frac{dw}{dt} = \frac{1}{3}w \]
which has the solution
\[ w(t) = e^{t/3} \]
for \( w(0) = 1 \).

2. The population at time \( t \) is
\[ p(t) = p(0)e^{0.05t} \]
\( p(20) = 2.718 \) million.

3. This is the same as the differential equation
\[ \frac{dp}{dt} = kp \]
with solution
\[ p(t) = p(0)e^{kt} \]
Given \( p(0) = 2 \) and \( p(45) = p(0)e^{45k} = 4 \) we find
\[ k = \frac{\ln 2}{45} = 0.0154 \]
In 1960, \( p(30) = 3.175 \) billion. If the population reaches 6 billion at time \( T \), we solve \( 6 = 2e^{kT} \) to get \( T = 71.32 \), i.e. in 2001.

4. \( t = \frac{\ln 12}{0.09} = 27.6 \)

5. 75000 years.

6. 17.3 days.

7. \( t = -\frac{\ln 0.3}{0.00012} = 1000 \) years.

8. The solution of the differential equation is

\[
y(t) = 1000 - 800e^{-0.1t}
\]

To reach a population of 500

\[
t = -\frac{\ln 5/8}{0.1} = 4.7
\]

9. Assignment question.

10. The solution of the differential equation is

\[
y(t) = 10^6(3 - 2e^{-kt})
\]

Using \( y(5) = 1.1 \times 10^6 \), we find

\[
k = -\frac{\ln 0.95}{5} = 0.0103
\]

The predicted population for 1990 is 1.1 million (the same as the data!). The population will reach 2 million in 2053. Although the population approaches 3 million as \( t \to \infty \), it will never actually reach this number.

11. (a) \( P(t) = \frac{100}{A - t} \)

(b) \( A = 10 \)

(c) \( P(t) \to \infty \) as \( t \to 10 \).

12. 18.3 months. (See 13 below for method of solution).

13. The solution of the differential equation is

\[
y(t) = \frac{AL}{A + e^{-kLt}}
\]

with \( k = 0.002 \) and \( L = 2000 \). The initial condition \( y(0) = 2 \) gives

\[
2 = \frac{AL}{A + 1}
\]
Solving for $A$ gives $A = 1/999$. Let $T$ be the time such that $y(T) = 1500$, then

$$1500 = \frac{AL}{A + e^{-kLT}}$$

Solving for $T$ we find

$$T = -\frac{\ln A/3}{KL} = 20 \text{ weeks}$$

14. $W(t) = 5e^{2.5(1-e^{-0.08 t})}$. The final weight is 60.91.

15. Assignment question.

Chapter 9

1. $\frac{\partial f}{\partial x} = 2xy^4$, $\frac{\partial f}{\partial y} = 4x^2y^3$

2. $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 - y^2}}$, $\frac{\partial f}{\partial y} = -\frac{y}{\sqrt{x^2 - y^2}}$

3. $\frac{\partial f}{\partial x} = e^{x+3y}$, $\frac{\partial f}{\partial y} = 3e^{x+3y}$

4. $\frac{\partial f}{\partial x} = y\cos(xy)$, $\frac{\partial f}{\partial y} = x\cos(xy)$

5. $\frac{\partial f}{\partial x} = \frac{2x}{x^2 + y^2}$, $\frac{\partial f}{\partial y} = \frac{2y}{x^2 + y^2}$

6. $\frac{\partial f}{\partial x} = -\frac{2x}{y}\sin(x^2)$, $\frac{\partial f}{\partial y} = -y^{-2}\cos(x^2)$

2. Both $= 24x^3y + 14x$.

3. $\frac{\partial f}{\partial x} = 4x + y$, $\frac{\partial f}{\partial y} = x + 2y$

Solving for the stationary points gives $x = 0, y = 0$. The Hessian matrix is

$$H = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$$

and det $H = 7 > 0$. Since $\frac{\partial^2 f}{\partial x^2} = 4 > 0$ the stationary point is a local minimum.

2. The only stationary point is $x = 0, y = 0$ which is a saddle point.

3. The partial derivatives are

$$\frac{\partial f}{\partial x} = 4x + 4x^3$$

Solving for the stationary points gives $x = 0$, and $y = 0, y = -1, y = 1$. The Hessian matrix is

$$H = \begin{bmatrix} 4 + 12x^2 & 0 \\ 0 & -4 + 12y^2 \end{bmatrix}$$

We find that $x = 0, y = 0$ is a saddle point and the other two stationary points are local minima.

3. The constrained maxima occur at i. $(2, -1, 3)$ (Lagrange multiplier $t = -2$)

ii. $(8, -3, 31)$ (Lagrange multiplier $t = 13$)