MATH110 — Answers To The Exercises

Chapter 1

1. All vectors have dimension 5.
2. $Z_1 = 0$, $Z_4 = -2$, $Z_5 = 1$.
3.

$$\begin{bmatrix} 0 \\ 5 \\ -2 \\ 11 \\ -4 \end{bmatrix}$$

4.

$$\begin{bmatrix} 0 \\ 9 \\ 0 \\ -6 \\ 3 \end{bmatrix}$$

5.

$$\begin{bmatrix} -1/2 \\ 3/2 \\ -5/2 \\ 7/2 \\ -9/2 \end{bmatrix}$$

6. 2.
7. -14.
8. $\sum_{i=1}^{5} Y_i^2 = 165$, $(\sum_{i=1}^{5} Y_i)^2 = 25$.
9.$< \mathbf{X}, \mathbf{X} >= \sum_{i=1}^{n} X_i^2$. Since each term $X_i^2 \geq 0$ the sum is ≥ 0.

Chapter 2

1. All are 4×3 matrices.
2. $A_{12} = 2$, $A_{23} = -3$ and $A_{41} = 3$.
3.

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -6 \\ 2 & 2 & -2 \\ -1 & -3 & -2 \end{bmatrix}$$

4.

$$2\mathbf{C} = \begin{bmatrix} -6 & -4 & 4 \\ 2 & 6 & 10 \\ -4 & -2 & 0 \\ -2 & -2 & -4 \end{bmatrix}$$

5.

$$\mathbf{A}+\mathbf{B}-2\mathbf{C}=\begin{bmatrix} 7 & 6 & -1 \\ -1 & -6 & -16 \\ 6 & 4 & -2 \\ 1 & -1 & 2 \end{bmatrix}$$

6.

$$\mathbf{B}=\mathbf{A}-\mathbf{C}=\begin{bmatrix} -1 & -3 & 1 & 6 \\ -4 & -2 & 10 & 5 \end{bmatrix}$$

7. $\mathbf{B}-\mathbf{A}=$ the increase in mosquito populations over the year.

$$\mathbf{B}-\mathbf{A}=\begin{bmatrix} 1262 & 709 & 70 & 26 \\ 1170 & 653 & 59 & 57 \\ 211 & 337 & 26 & 20 \end{bmatrix}$$

8.

$$\frac{1}{2}(\mathbf{B}-\mathbf{A})=\begin{bmatrix} 631.5 & 354.5 & 35.0 & 13.0 \\ 585.0 & 326.5 & 29.5 & 28.5 \\ 105.5 & 168.5 & 13.0 & 10.0 \end{bmatrix}$$

9.

$$\text{Total}=\mathbf{A}+\mathbf{B}=\begin{bmatrix} 153 & 82 & 33 \\ 43 & 68 & 55 \\ 56 & 60 & 37 \end{bmatrix}$$

10.

$$\text{Change}=\mathbf{A}-\mathbf{B}=\begin{bmatrix} -43 & 4 & -11 \\ -5 & 38 & 25 \\ -28 & 16 & 3 \end{bmatrix}$$

11.

$$\text{Total}=\frac{1}{2}(\mathbf{A}+\mathbf{B})=\begin{bmatrix} 76.5 & 41.0 & 16.5 \\ 21.5 & 34.0 & 27.5 \\ 28.0 & 30.0 & 18.5 \end{bmatrix}$$

12.

$$\mathbf{A}=\begin{bmatrix} 300 & 217 & 64 \\ 271 & 201 & 83 \end{bmatrix}$$

13.

$$\mathbf{B}=\begin{bmatrix} 284 & 196 & 58 \\ 288 & 202 & 69 \end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} 184 & 112 & 49 \\ 318 & 213 & 73 \end{bmatrix}$$

14. (i)

$$\mathbf{A}+\mathbf{B}=\begin{bmatrix} 584 & 413 & 122 \\ 559 & 403 & 152 \end{bmatrix}$$

(ii)

$$\mathbf{C}-\mathbf{B}=\begin{bmatrix} -100 & -84 & -9 \\ 30 & 11 & 4 \end{bmatrix}$$

(iii)

$$\frac{1}{2}(\mathbf{B}+\mathbf{C}) = \begin{bmatrix} 234.0 & 154.0 & 53.5 \\ 303.0 & 207.5 & 71.0 \end{bmatrix}$$

(iv)

$$\frac{1}{3}(\mathbf{A}+\mathbf{B}+\mathbf{C}) = \begin{bmatrix} 256.00 & 175.00 & 57.00 \\ 292.33 & 205.33 & 75.00 \end{bmatrix}$$

(v)

$$\frac{2}{3}\mathbf{A} - \frac{1}{3}\mathbf{B} - \frac{1}{3}\mathbf{C} = \begin{bmatrix} 44.00 & 42.00 & 7.00 \\ -21.33 & -4.33 & 8.00 \end{bmatrix}$$

15. (i) $\mathbf{A}+\mathbf{B} =$ Total production on the first two days.
(ii) $\mathbf{C}-\mathbf{B} =$ Increase in production from day 2 to day 3.
(iii) $\frac{1}{2}(\mathbf{B}+\mathbf{C}) =$ Average production on the second and third days.
(iv) $\frac{1}{3}(\mathbf{A}+\mathbf{B}+\mathbf{C}) =$ Average production over the three days.
(v) $\frac{2}{3}\mathbf{A}-\frac{1}{3}\mathbf{B}-\frac{1}{3}\mathbf{C} = \mathbf{A}-\frac{1}{3}(\mathbf{A}+\mathbf{B}+\mathbf{C}) =$ Excess of production on day 1 over the average production for the three days.
16. $\mathbf{A}$ is 2×3, $\mathbf{B}$ is 2×2 and $\mathbf{C}$ is 3×1.
17. The products $\mathbf{A}\times\mathbf{C}$, $\mathbf{B}\times\mathbf{A}$ and $\mathbf{B}\times\mathbf{B}$ are defined.
18.

$$\mathbf{A}\times\mathbf{C} = \begin{bmatrix} 8 \\ -7 \end{bmatrix}$$

$$\mathbf{B}\times\mathbf{A} = \begin{bmatrix} -2 & 7 & 2 \\ 2 & -10 & -3 \end{bmatrix}$$

$$\mathbf{B}\times\mathbf{B} = \begin{bmatrix} 3 & -8 \\ -4 & 11 \end{bmatrix}$$

19.

$$\mathbf{AC} = \begin{bmatrix} 2816 \\ 954 \\ 1430 \end{bmatrix}$$

This is the total amount of vegetation eaten in the three areas in 1958.
20.

$$(\mathbf{A}+\mathbf{B})\mathbf{C} = \begin{bmatrix} 4687.5 \\ 2860.5 \\ 2693.0 \end{bmatrix}$$

21.

$$\mathbf{AB} = \begin{bmatrix} 2200 & 3000 \\ 2750 & 3700 \end{bmatrix}$$

22. This is the cost of removing pollutants from products at the plants. The rows correspond to the products, the columns the plants.
23.

$$\mathbf{L} = \begin{bmatrix} 0 & 30 & 30 & 40 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}$$

24.

$$\mathbf{P_1} = \mathbf{LP} = \begin{bmatrix} 2000 \\ 0 \\ 10 \\ 10 \end{bmatrix}$$

25.

$$\mathbf{P_2} = \mathbf{LP_1} = \begin{bmatrix} 700 \\ 1800 \\ 0 \\ 5 \end{bmatrix}$$

$$\mathbf{P_3} = \mathbf{LP_2} = \begin{bmatrix} 54200 \\ 630 \\ 900 \\ 0 \end{bmatrix}$$

$$\mathbf{P_4} = \mathbf{LP_3} = \begin{bmatrix} 45900 \\ 48780 \\ 315 \\ 450 \end{bmatrix}$$

26.

$$\mathbf{A}^2 = \begin{bmatrix} 1 & -6 \\ 0 & 4 \end{bmatrix}$$

27.

$$\mathbf{A}^3 = \begin{bmatrix} 1 & -14 \\ 0 & 8 \end{bmatrix}$$

$$\mathbf{A}^4 = \begin{bmatrix} 1 & -30 \\ 0 & 16 \end{bmatrix}$$

28.

$$\mathbf{A}^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{bmatrix}$$

29.

$$\text{Both} = \begin{bmatrix} 1 & 1\frac{1}{2} \\ 0 & \frac{1}{4} \end{bmatrix}$$

30. Just check that $\mathbf{LL}^{-1} = \mathbf{I}$.

31.

$$\mathbf{P_{-1}} = \mathbf{L}^{-1}\mathbf{P} = \begin{bmatrix} 222.22 \\ 0 \\ 10 \\ 10 \end{bmatrix}$$

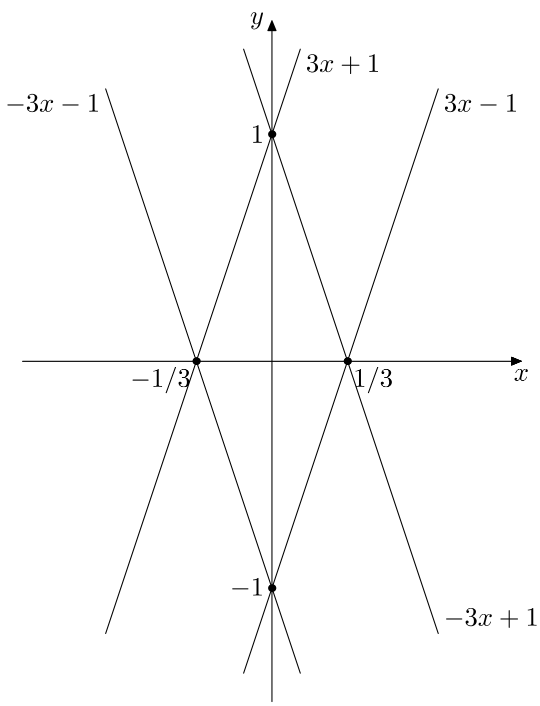
32.

$$\mathbf{P_{-2}} = \mathbf{L}^{-1}\mathbf{P_{-1}} = \begin{bmatrix} 0 \\ 20 \\ 20 \\ -24.44 \end{bmatrix}$$

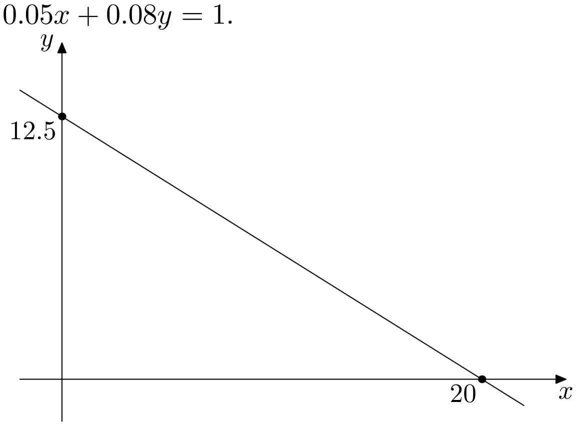
$$\mathbf{P_{-3}} = \mathbf{L}^{-1}\mathbf{P_{-2}} = \begin{bmatrix} 22.22 \\ 40 \\ -48.89 \\ 6.67 \end{bmatrix}$$

Chapter 3

1.



2. **(i)** $y = \frac{1}{5}x + 7$.
(ii) $y = 8x$.
3. $y = -2x + 11$.
4. The approximate equations are $y = 6x - 30$ and $y = -3.3x + 10$.
5. $y = -1.5x + 3$.
6. $0.05x + 0.08y = 1$.



7. **(i)** $y = 0.6x - 10$.

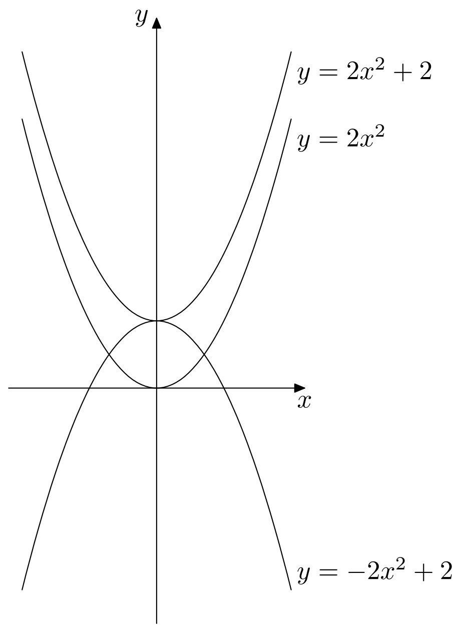
(ii) $b =$ rate of change of respiratory rate with partial pressure of CO_2.

(iii) 17.

(iv) -4. A negative respiratory rate is impossible.

(v) No. It may give a good description under normal conditions, but at low levels of CO_2 it is obviously wrong as the answer to **(iv)** shows.

8.



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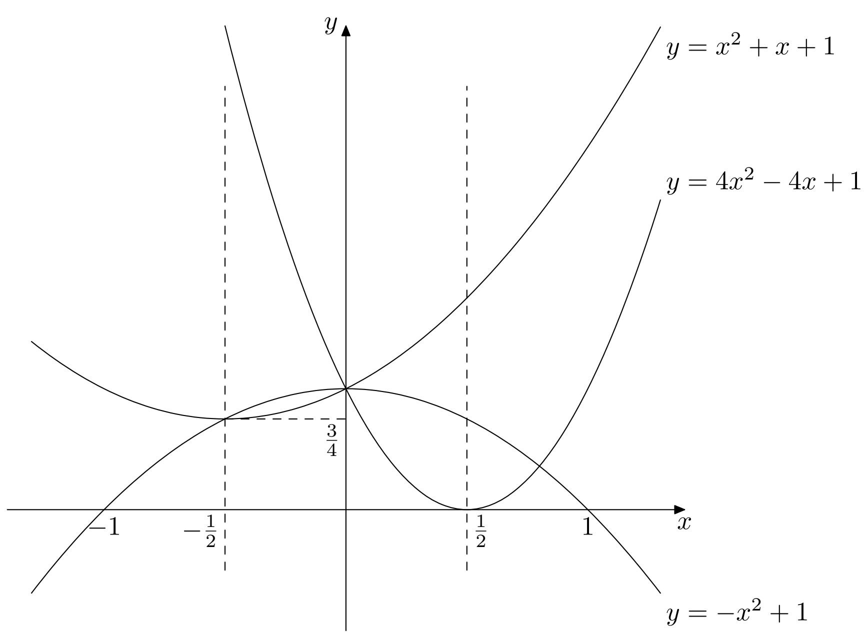
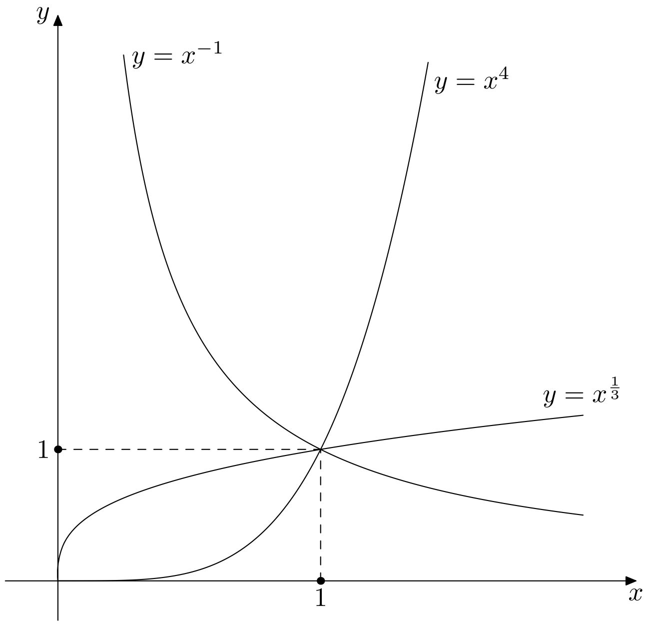


10. Yield $= x(500 - 5x) = -5x^2 + 500x$. The maximum yield occurs on the axis of symmetry of the quadratic, i.e. at $x = 50$, giving a maximum yield of 12500 lb per acre.

11. (i) 64, (ii) 8, (iii) 1, (iv) $8^{2/3} = \sqrt[3]{8^2} = 4$, (v) 1/8, (vi) 1/64, (vii) $8^{-1/3} = 1/\sqrt[3]{8} = 1/2$.

12.



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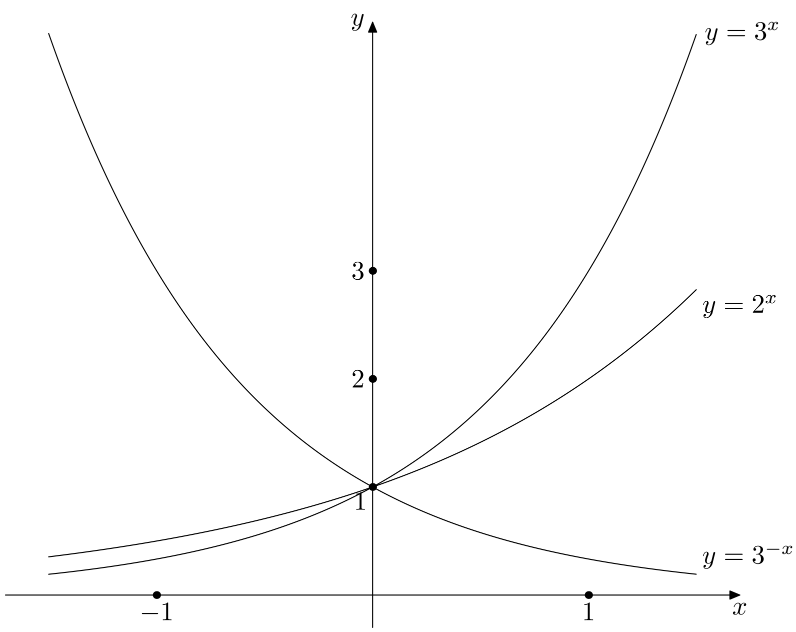
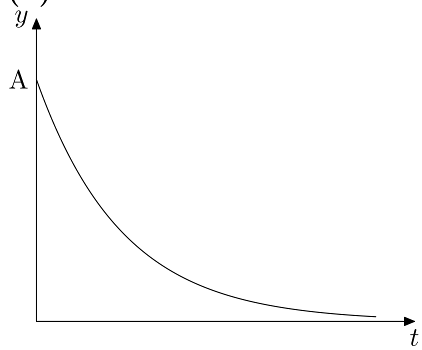


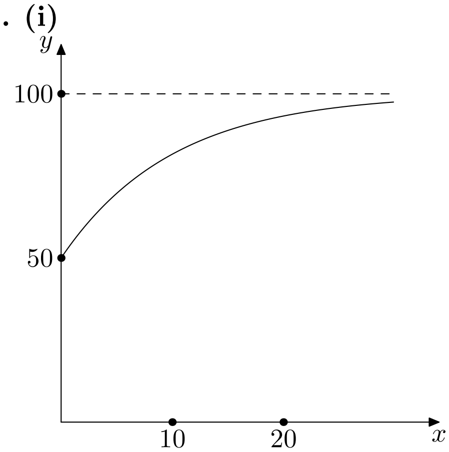
14. **(i)** $t = 0$ gives initial amount $= A$.

(ii)



(iii) $y \to 0$.

15. **(i)**



(ii) $y \to 100$.

(ii) *Very* large amounts of fertilizer would probably kill the crop.

16.

$$\begin{aligned}
\ln 6 &= \ln(2.3) = \ln 2 + \ln 3 = 1.792 \\
\ln 64 &= \ln(2^6) = 6 \ln 2 = 4.158 \\
\ln(\frac{1}{128}) &= \ln(2^{-7}) = -7 \ln 2 = -4.851 \\
\ln(\frac{8}{9}) &= \ln(\frac{2^3}{3^2}) = 3 \ln 2 - 2 \ln 3 = -0.119
\end{aligned}$$

17. **(i)** (a) 3.51 (b) 9.68

(ii) 10^{-3}

18. **(i)** 10^3

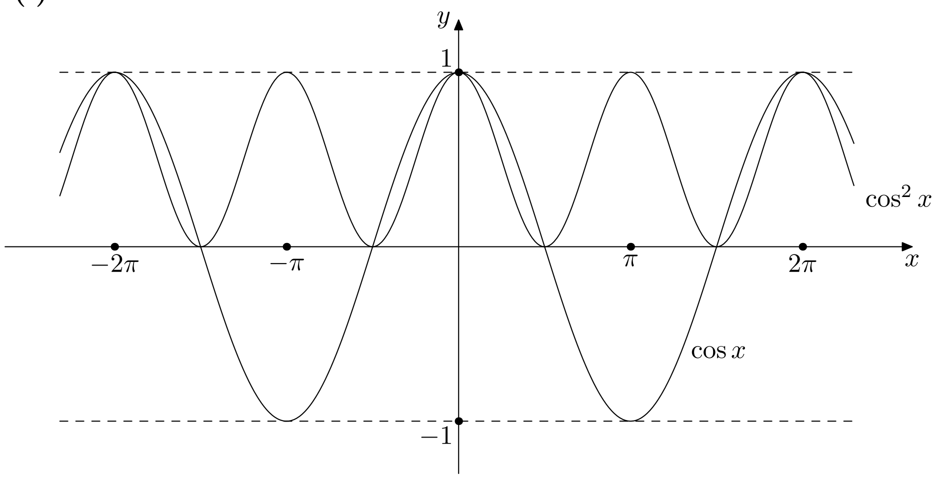
(ii) $8.9 - \log 4 = 8.30$

(iii) $10^{(8.9-6.23)} = 470$

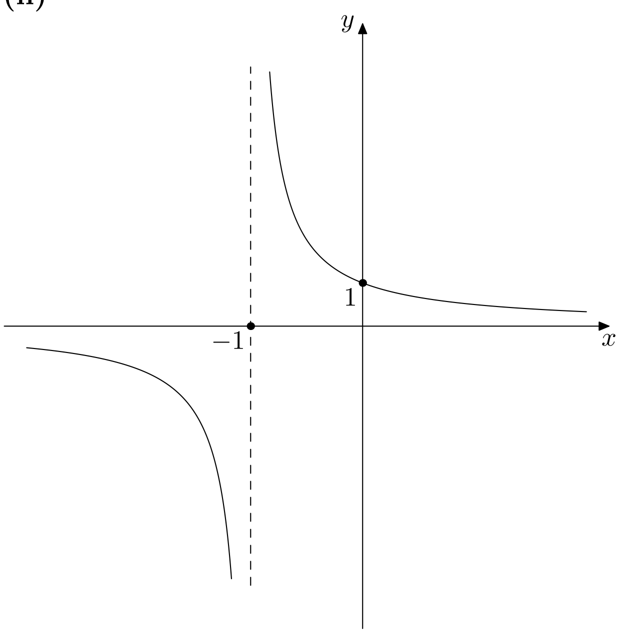
19. height $= 7 \tan 23 = 2971m$

20. (i) 1/2, (ii) 1, (iii) -1/2, (iv) -1, (v) -1, (vi) 0, (vii) -1, (viii) 1

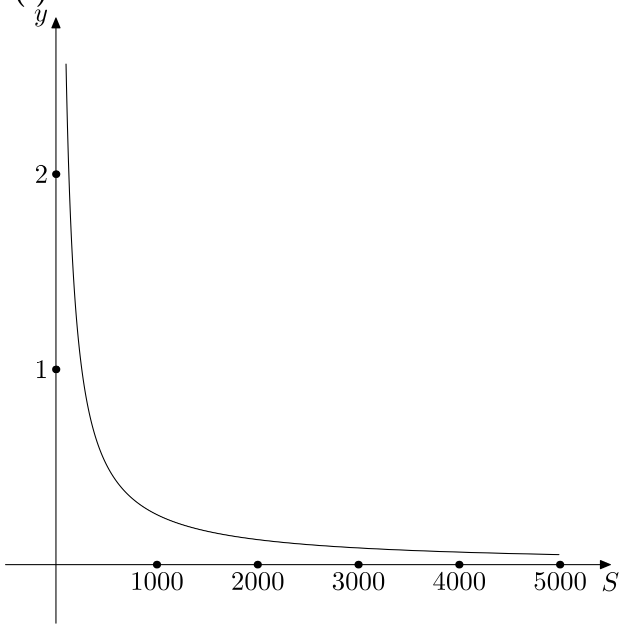
21. (i)



(ii)



22. (i)



(ii) (a) 0.1047, (b) 0.2035, (c) 51.28

(iii) 512.8

(iv) No. There is a limit to the number of trees a given area of land can support, so this would indicate that the relationship is incorrect for large S, although in practical terms the yield is so low it doesn't matter. For small S the relationship predicts arbitrarily large yields which is inconsistent with there being an upper limit to the yield per tree.

Chapter 4

1. $x = 1$
2. $x = \frac{-3 \pm \sqrt{17}}{2}$
3. $x = 1.5^{2/3} = 1.3104$
4. $x = \frac{\ln 2}{2} = 0.3466$
5. $x = \frac{\pi}{18}$ (and other answers)
6. Eliminating y gives $x = -5\frac{1}{2}$ and then $y = -10\frac{1}{2}$.
7. Elimninating y gives

$$x^2 - 4x + 4 = 0$$

and solving gives $x = 2$, $y = -2$.

Chapter 5

1. The average rate of change is

$$\frac{p(7) - p(2)}{5} = 0.4$$

2. The average rate of change is

$$\frac{f(1 + \Delta x) - f(1)}{\Delta x} = \frac{\Delta x + 2\Delta x^2}{\Delta x} = 1 + 2\Delta x$$

Then

$$f'(1) = \lim_{\Delta x \to 0} (1 + 2\Delta x) = 1$$

3. The average rate of change is

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{-3\Delta x + 4x\Delta x + 2\Delta x^2}{\Delta x} = -3 + 4x + 2\Delta x$$

Then

$$f'(x) = \lim_{\Delta x \to 0} (-3 + 4x + 2\Delta x) = -3 + 4x$$

4. The definition of the derivative gives $f'(3) = -14$ and the equation of the tangent is $y = -14x + 30$.
5. **(a)** $y' = 60x^5$
(b) $y' = 24x^7 + 30x^4 - 8x$
(c) $y' = 7.5t^{6.5}$
(d) $y = x^2 + 2 - 3x^{-3}$, so $y' = 2x + 9x^{-4}$
(e) $u' = -15t^{-4} + 8t$
(f) $y = x^{7/6}$, so $y' = \frac{7}{6}x^{1/6}$
(g) Expanding gives $v = 2x^3 + 2x^2 - 12x$ and $v' = 6x^2 + 4x - 12$.
6. $y = 8x - 1$.
7. **(a)** Product rule. $y' = (2x - 4)(x^3 + 4) + (x^2 - 4x + 3)3x^2$
(b) Chain rule. $y' = \frac{7}{3}(4x^3 + 9x^2)(x^4 + 3x^3 + 1)^{-2/3}$
(c) Chain rule. $y' = 3x(x^2 + 7)^{-1/2}$

(d) Quotient rule. $y' = \frac{2x(3x^3+4)-9x^2(x^3-3)}{(3x^3+4)^2}$

(e) $y = 2x^{3/2} + x^{-1/2}$, $y' = \frac{3}{4}x^{1/2} - \frac{1}{2}x^{-3/2}$

8. $u = \dfrac{t^2+4}{2t^2-3t}, \quad u' = \dfrac{2t(2t^2-3t)-(4t-3)(t^2+4)}{(2t^2-3t)^2}$

9. **(i)** m_0 is the mass at $t = 0$.

(ii) $\frac{dm}{dt} = (\frac{1}{3}t + m_0^{1/3})^2$

(iii) $m_0^{2/3}$, $(\frac{1}{3} + m_0^{1/3})^2$

(iv) No. Since $\frac{dm}{dt}$ is a square it is never negative, the mass cannot decrease.

10. $\frac{dy}{dx} = 15x^4 + 21x^2 - 8x$ $\frac{d^2y}{dx^2} = 60x^3 + 42x - 8$, $\frac{d^6y}{dx^6} = 0$.

11. **(a)** $12(t+1)^{-4}$

(b) $(t^2+1)^{-3/2}$

12. **(a)** The velocity is $s' = 3 - 2t$ which is zero at $t = \frac{3}{2}$.

(b) The acceleration is $s'' = 2$ (the same for all t).

13. **(a)** Chain rule. $y' = 2xe^{x^2}$

(b) Product rule. $y = x^{-1}e^x$, $y' = -x^{-2}e^x + x^{-1}e^x$

(c) Chain rule. $y = e^{\sqrt{x}-x}$, $y' = (\frac{1}{2}x^{-1/2} - 1)e^{\sqrt{x}-x}$

(d) $y = \ln(x^4) = 4\ln x$, $y' = 4/x$

(e) Chain rule. $y' = \frac{5}{x}(\ln x)^4$

(f) Product rule. $y' = \frac{1}{x}e^x + \ln x e^x$

(g) Product and chain rules. $y' = 2x\ln(x^2+x) + \frac{x^2(2x+1)}{x^2+x}$

(h) $y = \ln e^x + \ln(x-1)^{1/2} - \ln x^3 = x + \frac{1}{2}\ln(x-1) - 3\ln x$, $y' = 1 + \frac{1}{2}\frac{1}{x-1} - \frac{3}{x}$

14. $y' = 3ckpe^{-kt}(1-ce^{-kt})^2$

15. **(a)** $y' = 3\cos(3x)$

(b) $y' = -x^{-2}\cos x - x^{-1}\sin x$

(c) $y' = x^{-1/2}\sin(\sqrt{x})\cos(\sqrt{x})$

(d) $y' = \frac{15}{2}x^2(x^3+1)^{-1/2}\cos x - 5\sqrt{x^3+1}\sin x$

(e) $y' = -2\sin(2x)e^{\cos(2x)}$

(f) $y' = -\frac{\sin x}{\cos x}$

(g) $y' = -\frac{1+\cos x}{\sin^2 x}$ (using $\sin^2 x + \cos^2 x = 1$)

16. **(a)** $y'' = 2\cos x - x\sin x$

(b) $y'' = 2(\cos^2 x - \sin^2 x)$

17. The rate of growth is

$$w' = e^t(2 - \cos t + \sin t)$$

$w'(0) = e$, $w'(\frac{\pi}{2}) = 3e^{\pi/2}$, $w'(2\pi) = e^{2\pi}$. w can never decrease, since w' is never negative. To see this note that $\sin x$ and $\cos x$ are never greater than 1 in magnitude, so the term in brackets in the formaula for w' is never negative.

Chapter 6

1. **(a)** $x = 1, y = -22$, local minimum; $x = -2, y = 5$, local maximum.

(b) $x = \frac{1}{2}, y = \frac{1}{2}e^{-1}$, local maximum.

2. The maximum at $t = 1$ is $\frac{1}{20}$.

3. The only stationary point is a local minimum at $t = 4$. This indicates that the number of bacteria at first decrease, but then increase again after 4 hours. Further application of the drug will be neceassary to permanently decrease the bacteria count.
4. $R = \frac{2}{3}R_0$
5. The maximum of $A(\frac{5}{6})^{2.5}e^{-2.5}$ occurs at $t = \frac{5}{6}$.

Chapter 7

1. **(a)** $\int x^7 dx = \frac{1}{8}x^8$
(b) $\int x^{1/2} dx = \frac{2}{3}x^{3/2}$
(c) $\int (x^{-1/2} + 3x^{-2} dx = 2x^{1/2} - 3x^{-1}$
(d) $\int 2e^{-x/2} dx = -4e^{-x/2}$
(e) $\int (3\sin(x/2) + 2\cos(2x))dx = -6\cos(x/2) + \sin(2x)$
(d) $\frac{1}{8}x^8 + \frac{25}{7}x^2 - 7\ln x$
2. All of these require that that functions to be integrated are expanded before integration.
(a) $\int (\frac{4}{3}t + \frac{4}{3} + \frac{1}{3}t^{-1})dx = \frac{2}{3}t^2 + \frac{4}{3}t + \frac{1}{3}\ln t$
(b) $\int (\frac{2}{3}y^2 + \frac{7}{3}y - 2 + 3y^{-1})dy = \frac{2}{9}y^3 + \frac{7}{6}y^2 - 2y + 3\ln y$
(c) $\int (2x^2 + x - 1)dx = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x$
(b) $\int (t^{5/2} - 2t^{3/2} + t^{1/2})dt = \frac{2}{7}t^{7/2} - \frac{4}{5}t^{5/2} + \frac{2}{3}t^{3/2}$
3. (1a) $\leftrightarrow$ (2d), (1b) $\leftrightarrow$ (2a), (1c) $\leftrightarrow$ (2b), (1d) $\leftrightarrow$ (2c).
4. $\int_0^4 32t dt = 256 ft$.
5. The velocity is

$$\frac{dh}{dt} = \int \frac{d^2h}{dt^2} dt = -32t + c$$

The initial condition gives $c = 1000$. Now

$$h(t) = \int \frac{dh}{dt} dt = \int (-32t + 1000)dt = -16t^2 + 1000t$$

The maximum altitude occurs when $\frac{dh}{dt} = 0$, i.e $t = \frac{125}{4}$ and $h(t) = 15625$ ft. The bullet returns to earth when $h(t) = 0$, i.e. at $t = \frac{125}{2}$ secs.
6. The rate of blood flow is $v = 0.07(150 - 3t)$. Therefore the answer is

$$\int_0^{20} 0.07(150 - 3t)dt = 168 \text{ litres}$$

7. In each of these the function is positive on the interval of integration.
(a) Area $= \int_0^{\pi/2} \cos x dx = [\sin x]_0^{\pi/2} = 1$

(b) Area $= \int_{-2}^{2}(e^{2x}+7)dx = [\frac{1}{2}e^{2x}+7x]_{-2}^{2} = \frac{1}{2}(e^4-e^{-4})+28$

(c) Area $= \int_{1}^{8}(3x^{1/2}+x^{-1})dx = [2x^{3/2}+\ln x]_1^8 = 32\sqrt{2}+\ln 8-2$

8. **(a)** Integral = -9, Area = 9.

(b) Integral = 0, Area = 8.

(c) Integral = 2, Area = 6.

(d) Integral = $\frac{16}{3}$, Area = $\frac{17}{3}$.

9. **(a)** Area $= \int_{1}^{2}(5x-2x^2)dx = \frac{17}{6}$

(b) Area $= \int_{0}^{1}(e^x-x^2)dx = e-\frac{4}{3}$

10.

For drug A: $F_A(t) = \int 10e^{-t}dt = -10e^{-t}$

For drug B: $F_B(t) = \int(10e^{-t/2}-8e^{-2t}) = -20e^{-t/2}+4e^{-2t}$

(i) $F_A(1)-F_A(0) = 10(1-e^{-1}) = 6.32$,
$F_B(1)-F_B(0) = 16-20e^{-1/2}+4e^{-2} = 4.41$
Drug A is more effective.

(ii) $F_A(5)-F_A(0) = 10(1-e^{-5}) = 9.93$,
$F_B(5)-F_B(0) = 16-20e^{-5/2}+4e^{-10} = 14.36$
Drug B is more effective.

(iii) The averages from $t=0$ to $t=5$ are: drug A 1.99, drug B 2.87

Chapter 8

1. The differential equation is

$$\frac{dw}{dt} = \frac{1}{3}w$$

which has the solution

$$w(t) = e^{t/3}$$

for $w(0) = 1$.

2. The population at time t is

$$p(t) = p(0)e^{0.05t}$$

$p(20) = 2.718$ million.

3. This is the same as the differential equation

$$\frac{dp}{dt} = kp$$

with solution

$$p(t) = p(0)e^{kt}$$

Given $p(0) = 2$ and $p(45) = p(0)e^{45k} = 4$ we find

$$k = \frac{\ln 2}{45} = 0.0154$$

In 1960, $p(30) = 3.175$ billion. If the population reaches 6 billion at time T, we solve $6 = 2e^{kT}$ to get $T = 71.32$, i.e. in 2001.
4. $t = \frac{\ln 12}{0.09} = 27.6$
5. 75000 years.
6. 17.3 days.
7. $t = -\frac{\ln 0.3}{0.00012} = 1000$ years.
8. The solution of the differential equation is

$$y(t) = 1000 - 800e^{-0.1t}$$

To reach a population of 500

$$t = -\frac{\ln 5/8}{0.1} = 4.7$$

9. Assignment question.
10. The solution of the differential equation is

$$y(t) = 10^6(3 - 2e^{-kt})$$

Using $y(5) = 1.1 \times 10^6$, we find

$$k = -\frac{\ln 0.95}{5} = 0.0103$$

The predicted population for 1990 is 1.1 million (the same as the data!). The population will reach 2 million in 2053. Although the population approaches 3 million as $t \to \infty$ it will never actually reach this number.
11. **(a)** $P(t) = \dfrac{100}{A - t}$
(b) $A = 10$
(c) $P(t) \to \infty$ as $t \to 10$.
12. 18.3 months. (See **13** below for method of solution).
13. The solution of the differential equation is

$$y(t) = \frac{AL}{A + e^{-kLt}}$$

with $k = 0.002$ and $L = 2000$. The initial condition $y(0) = 2$ gives

$$2 = \frac{AL}{A + 1}$$

Solving for A gives $A = 1/999$. Let T be the time such that $y(T) = 1500$, then

$$1500 = \frac{AL}{A + e^{-kLT}}$$

Solving for T we find

$$T = -\frac{\ln A/3}{KL} = 20 \text{ weeks}$$

14. $W(t) = 5e^{2.5(1-e^{-0.08t})}$. The final weight is 60.91.
15. Assignment question.

Chapter 9

1. **1.** $\frac{\partial f}{\partial x} = 2xy^4$, $\frac{\partial f}{\partial y} = 4x^2y^3$

2. $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 - y^2}}$, $\frac{\partial f}{\partial y} = -\frac{y}{\sqrt{x^2 - y^2}}$

3. $\frac{\partial f}{\partial x} = e^{x+3y}$, $\frac{\partial f}{\partial y} = 3e^{x+3y}$

4. $\frac{\partial f}{\partial x} = y\cos(xy)$, $\frac{\partial f}{\partial y} = x\cos(xy)$

5. $\frac{\partial f}{\partial x} = \frac{2x}{x^2 + y^2}$, $\frac{\partial f}{\partial y} = \frac{2y}{x^2 + y^2}$

6. $\frac{\partial f}{\partial x} = -\frac{2x}{y}\sin(x^2)$, $\frac{\partial f}{\partial y} = -y^{-2}\cos(x^2)$

2. **1.** Both $= 24x^3y + 14x$.

2. Both $= \cos(xy) - xy\sin(xy) + \sin(xy) + xy\cos(xy)$

3. **1.** The partial derivatives are

$$\frac{\partial f}{\partial x} = 4x + y, \quad \frac{\partial f}{\partial y} = x + 2y$$

Solving for the stationary points gives $x = 0$, $y = 0$. The Hessian matrix is

$$H = \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$$

and $\det H = 7 > 0$. Since $\frac{\partial^2 f}{\partial x^2} = 4 > 0$ the stationary point is a local minimum.

2. The only stationary point is $x = 0$, $y = 0$ which is a saddle point.

3. The partial derivatives are

$$\frac{\partial f}{\partial x} = 4x + 4x^3, \quad \frac{\partial f}{\partial y} = -4y + 4y^3$$

Solving for the stationary points gives $x = 0$, and $y = 0$, $y = -1$, $y = 1$. The Hessian matrix is

$$H = \begin{bmatrix} 4 + 12x^2 & 0 \\ 0 & -4 + 12y^2 \end{bmatrix}$$

We find that $x = 0$, $y = 0$ is a saddle point and the other two stationary points are local minima.

3. The constrained maxima occur at **i.** $(2, -1, 3)$ (Lagrange multiplier $t = -2$)
ii. $(8, -3, 31)$ (Lagrange multiplier $t = 13$)