

Mathematics 110 — Assignment 2

Reading: Lecture Notes Sections 2.3 to 2.6

Question 1.

Let \mathbf{A} be the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 4 \\ 0 & 1 & 0 \end{bmatrix}$$

- (a) Calculate \mathbf{A}^2 and \mathbf{A}^3 .
- (b) Show that

$$\mathbf{A}^{-1} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & 1 \\ 0 & 0 & 1 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{2} \end{bmatrix}.$$

- (c) Suppose \mathbf{X} is a three dimensional vector and

$$\mathbf{A}\mathbf{X} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}.$$

Use \mathbf{A}^{-1} to find \mathbf{X} .

Question 2.

Let \mathbf{C} be the matrix

$$\mathbf{C} = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}.$$

- (a) Find the determinant of \mathbf{C} .
- (b) Find \mathbf{C}^{-1} .

Question 3.

A species lives for 4 years and its yearly birth and survival rates are described by the Leslie matrix

$$\mathbf{L} = \begin{bmatrix} 0 & 0.1 & 1 & 0.5 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \end{bmatrix}.$$

- (a) What are the birth rates for individuals aged 0–1 year, 1-2 years and 3-4 years?
- (b) What fraction of individuals aged 1-2 years survive to age 2-3 years? What fraction of individuals aged 2-3 years survive to age 3-4 years?
- (c) Calculate \mathbf{L}^2 .

(d) If the population is initially described by the vector

$$\mathbf{P} = \begin{bmatrix} 1000 \\ 1200 \\ 1100 \\ 600 \end{bmatrix}$$

calculate the population vector after 1 year.

(e) Use your answer to part (c) to calculate the population vector after 2 years.

(f) Check that the inverse of \mathbf{L} is given by

$$\mathbf{L}^{-1} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 1.25 & 0 \\ 0 & 0 & 0 & 1.25 \\ 2 & 0 & -0.25 & -2.5 \end{bmatrix}.$$

(g) Using the initial population vector \mathbf{P} in part (d), use \mathbf{L}^{-1} to find the population vector 1 year ago.