Sample Solutions for Tutorial 10

Question 1.

(a)

\[3x + 4y = 10 \quad (i)\]
\[7x - 5y = 9 \quad (ii)\]
\[7 \times (i) - 3 \times (ii) : 43y = 43 \quad (iii)\]
\[5 \times (i) + 4 \times (ii) : 43x = 86 \quad (iv)\]

Hence \[x = 2, y = 1.\]

(b)

\[3x + 4y = 10 \quad (i)\]
\[7x - 5y = 9 \quad (ii)\]
\[5x + 6y = 16 \quad (iii)\]
\[7 \times (i) - 3 \times (ii) : 43y = 43 \quad (iv)\]
\[5 \times (i) + 4 \times (ii) : 43x = 86 \quad (v)\]

It follows that \[x = 2, y = 1\] is the only possible solution.

We substitute this into (iii) and find

\[5 \times 2 + 6 \times 1 = 10 + 6 = 15\]

showing that (iii) is also satisfied. Hence the system has the unique solution \[x = 2, y = 1.\]

(c)

\[2x + 3y + z = 14 \quad (i)\]
\[3x + 6y - z = 8 \quad (ii)\]
\[2 \times (i) - (ii) : x + 3z = 20 \quad (iii)\]
\[2 \times (ii) - 3 \times (i) : 3y - 5z = -26 \quad (iv)\]

It follows that \[x = 20 - 3z, y = \frac{1}{3}(5z - 26).\]

Hence the system of equations has infinitely many solutions, one for each real number \(t\), namely, \(x = -9t + 20, y = 5t - \frac{20}{3}, z = 3t\)

(d)

\[a + b + c = 12 \quad (i)\]
\[4a + 2b + c = 23 \quad (ii)\]
\[9a + 3b + c = 40 \quad (iii)\]
\[(ii) - (i) : 3a + b = 11 \quad (iv)\]
\[(iii) - (i) : 8a + 2b = 28 \quad (v)\]
\[(v) - 2 \times (iv) : 2a = 6 \quad (vi)\]
\[(v) - \frac{3}{2} \times (vi) : b = 2 \quad (vii)\]
\[(i) - (vii) - \frac{1}{2} \times (vi) : c = 7 \quad (viii)\]

Hence the system of equations has the unique solution \(a = 3, b = 2, c = 7.\)

[The reader should verify that these are, in fact, solutions by substituting into the original systems of equations.]
Question 2.
Suppose that
\[ r = 2x + 3y \]
\[ s = 3x + 6y \]
with
\[ x = 3u + w \]
\[ y = 2u + 7v - 5w \]
Then
\[ r = 2(3u + w) + 3(2u + 7v - 5w) \]
\[ = (2 + 3.2)u + (2.0 + 3.7)v + (2.1 + 3.(-5))w \]
\[ = 12u + 21v - 13w \]
\[ s = 3(3u + w) + 6(2u + 7v - 5w) \]
\[ = (3 + 6.2)u + (3.0 + 6.7)v + (3.1 + 6(-5))w \]
\[ = 21u + 42v - 27w \]
Thus
\[ r = 12u + 21v - 13w \]
\[ s = 21u + 42v - 27w \]

Question 3.
Suppose that
\[ r = 2x + 3y + 4z \]
\[ s = 3x + 6y - z \]
\[ t = 2x + y + 6z \]
and
\[ u = + 3y + z \]
\[ v = 3x + 7y - 5z \]
\[ w = 2x - y + 3z \]
Then
\[ r + u = (2x + 3y + 4z) + (3y + z) \]
\[ = (2 + 0)x + (3 + 3)y + (4 + 1)z \]
\[ = 2x + 6y + 5z \]
\[ s + v = (3x + 6y - z) + (3x + 7y - 5z) \]
\[ = (3 + 0)x + (6 + 7)y + (-1 - 5)z \]
\[ = 3x + 13y - 6z \]
\[ t + w = (2x + y + 6z) + (2x - y + 3z) \]
\[ = (2 + 2)x + (1 - 1)y + (6 + 3)z \]
\[ = 4x + 9z \]
Thus
\[ r + u = 3x + 6y + 5z \]
\[ s + v = 3x + 13y - 6z \]
\[ t + w = 4x + 9z \]