ASSIGNMENT 5  
(Post–By Date: 20th April)

Question 1.

For each of the following functions, state at which points the function is differentiable and find the derivative. If a function is not differentiable at a point explain why.

(a) \( f : \mathbb{R} \rightarrow \mathbb{R}, \ x \mapsto 5x^4 - 3x^3 + x^2 - 1 \)

(b) \( g : \mathbb{R} \rightarrow \mathbb{R}, \ x \mapsto \frac{2 - x^2}{2 + x^2} \)

(c) \( h : \mathbb{R} \rightarrow \mathbb{R}, \ x \mapsto \begin{cases} x^2 + 2x & \text{if } x \leq 0 \\ \frac{x}{1 + x} & \text{if } x > 0 \end{cases} \)

(d) \( j : \mathbb{R} \rightarrow \mathbb{R}, \ x \mapsto \sqrt{4x^2 + 1} \)

Question 2.

Use the formal definition of the derivative to prove that, for \( x > 0 \),

\[
\frac{d}{dx} (\sqrt{x}) = \frac{1}{2\sqrt{x}}.
\]

Question 3.

Differentiate the following functions twice, where possible. State where each function or its derivative, fails to be differentiable.

(a) \( f : \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}, \ x \mapsto \frac{x^2}{1 - x} \)

(b) \( g : \mathbb{R} \rightarrow \mathbb{R}, \ u \mapsto \sqrt{u^4 + 1} \)

(c) \( h : \mathbb{R} \setminus \{\pm 1\} \rightarrow \mathbb{R}, \ x \mapsto (1 - x^4)^{-2} \)

(d) \( j : \mathbb{R} \rightarrow \mathbb{R}, \ t \mapsto t^7 - 5t^4 + 3 \)

Question 4 is on the next page.
Question 4.

Take the function
\[ f : \mathbb{R} \rightarrow \mathbb{R}, \quad x \mapsto \frac{1}{4 + x^2} \]

(a) Find the points where the tangent line to the graph of \( f \) is horizontal.

(b) Discuss the behaviour of \( f \) near the points of part (a) and as \( x \to \pm \infty \).

(c) Sketch the graph of \( f \).

Question 5\(^*\) (Optional).

Prove, formally, that if the function \( f : \mathbb{R} \to \mathbb{R} \) is differentiable and if \( f(x) \neq 0 \) for all \( x \in \mathbb{R} \), then
\[ g : \mathbb{R} \to \mathbb{R}, \quad u \mapsto \frac{1}{f(u)} \]

is also a differentiable function and for all \( u \in \mathbb{R} \),
\[ \frac{d}{du} \left( \frac{1}{f(u)} \right) = -\frac{f'(u)}{(f(u))^2}. \]