ASSIGNMENT 3  
(Post–By Date: 23rd March)

Question 1.

For each of the formulæ below, find the maximum subset $X$, of $\mathbb{R}$ such that the given formula can be used to define a function, $f: X \to \mathbb{R}$. For each of these functions state whether or not the function is injective, surjective or bijective. Justify your answers.

(a) $f(x) = \frac{1}{1 + x}$

(b) $f(x) = \sqrt{x^4 - x^2}$

(c) $f(x) = x^3 - 13$

Question 2.

Show that given $\varepsilon > 0$ there is a $\delta > 0$ such that $\left| \frac{1}{1+x^2} - 1 \right| < \varepsilon$ whenever $|x| < \delta$.

Question 3.

Evaluate the following limits.

(a) $\lim_{x \to \infty} \frac{x^4 + 2x - 1}{3x^4 + x^3}$

(b) $\lim_{x \to 1} \frac{\sqrt{3x - 1} - \sqrt{x + 3}}{x}$

(c) $\lim_{x \to 0} \frac{\tan x}{x}$

(d) $\lim_{x \to 0} \frac{\cos^2 x - \cos x}{x}$

Question 4.

Prove, formally, that $\lim_{x \to 1} x^3 = 1$.

Question 5 is on the next page.
Question 5.

For $k \in \mathbb{R}$ consider the function

$$f : \mathbb{R} \rightarrow \mathbb{R}, \quad x \mapsto \begin{cases} 
\frac{1}{1 + x^4} & \text{if } x < 1 \\
1 + x^4 & \text{if } x \geq 1,
\end{cases}$$

(a) Which, if any, value(s) of $k$ render $f$ continuous at $x = 1$?

(b) Show that $f$ is then continuous on $]0, \infty[$.