ASSIGNMENT 10

(Post–By Date: 25th May)

**Question 1.**

Evaluate the following determinants. If you use any of the properties of determinants, state concisely which property you have used.

\[
\begin{vmatrix}
2 & 0 & -1 & 0 \\
1 & 1 & 1 & 1 \\
-2 & 3 & 1 & 0 \\
0 & 4 & 2 & -1 \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
-15 & 12 & 27 & 1 \\
0 & 2 & 0 & 15 \\
0 & 0 & 1 & 10 \\
0 & 0 & 0 & 3 \\
\end{vmatrix}
\]

**Question 2.**

Find the eigenvalues and associated eigenvectors for the matrix \[
\begin{bmatrix}
2 & 7 \\
7 & 2 \\
\end{bmatrix}
\].

**Question 3.**

Let \( P, Q, R \) and \( S \) be the four points in \( \mathbb{R}^3 \) with co–ordinates \((-1, 0, 0), (1, 0, 0), (1, 1, 1) \) and \((2, 1, 2)\), respectively.

(a) Write \( \vec{PQ} \) and \( \vec{PR} \) in terms of the standard unit vectors \( i, j \) and \( k \).

(b) Find the orthogonal projection of \( \vec{PQ} \) onto \( \vec{PR} \).

(c) Find the area of the triangle with vertices \( P, Q \) and \( R \).

(d) Find the volume of the parallelepiped with sides given by the vectors \( \vec{PQ}, \vec{PR} \) and \( \vec{PS} \).

Question 4 is on the next page.
Let $A$ and $B$ be distinct points in $\mathbb{R}^3$. Show that the distance, $d$, of the point $P \in \mathbb{R}^3$ from the line through $A$ and $B$ is given by

$$d = \frac{\|\vec{PA} \times \vec{PB}\|}{\|\vec{AB}\|}.$$