Question 1.

Prove, by induction, that for all counting numbers $n$,

$$
\sum_{k=1}^{n} (2k - 1) = 1 + 3 + 5 + \ldots + (2n - 1) = n^2.
$$

Question 2.

Use induction to prove that for every natural number, $n$, $3^{3n} - 1$ is divisible by 26.

Question 3.

Let $A$ and $B$ be two sets. Prove the following propositions are logically equivalent.

(i) $B \subseteq A$

(ii) $A \cap B = B$

(iii) $A \cup B = A$

Question 4.

Given any subset $Y$ of the set $X$, put $Y' := X \setminus Y = \{x \in X \mid x \notin Y\}$.

Use Venn diagrams to demonstrate that, for all subsets $A$ and $B$ of $X$,

$$(A \cup B)' = A' \cap B'.$$

Question 5.

Give both a direct proof and a proof by contradiction of the statement

If $x$ is a real number and $x > 1$, then $x^2 > 1$. 