Recurrence Relations
Simplest Case of General Solutions

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Outline

1. Simplest Case of General Solutions
   - Example
Simplest Case of General Solutions

\[ c_m a_{n+m} + c_{m-1} a_{n+m-1} + \cdots + c_1 a_{n+1} + c_0 a_n = g(n), \quad n \geq 0 \quad (***) \]

\[ \sum_{k=0}^{m} c_k a_{n+k} = g(n) \]

**Definition**

A solution \( a_n \) of a recurrence relation (***) is said to be a **general solution**, typically containing some arbitrary constants in the solution expression for \( a_n \), if any **particular solution** of the recurrence relation (***) can be obtained as a **special case** of the general solution.
Verify that $a_n = A2^n + B3^n$ for arbitrary constants $A$ and $B$ solves the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 0$.

Likewise we can show that $a_n = 5 \times 2^n$ is also a (particular) solution. Obviously the particular solution $a_n = 5 \times 2^n$ is included in the more general solution expression $a_n = A2^n + B3^n$ if we choose $A = 5$ and $B = 0$.

In fact one can show that all the solutions of $a_{n+2} - 5a_{n+1} + 6a_n = 0$ are embraced by the solution expression $a_n = A2^n + B3^n$, which is hence the general solution.
Simplest Case of General Solutions

An alternative way to determine if a solution is the general solution of an \( m \)-th order linear, constant coefficient recurrence relation is to see if the solution expression contains exactly \( m \) independent arbitrary constants.
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- An alternative way to determine if a solution is the general solution of an $m$-th order linear, constant coefficient recurrence relation is to see if the solution expression contains exactly $m$ independent arbitrary constants.

- The word *independent* here roughly means that none of the arbitrary constants can be made redundant.
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- The word independent here roughly means that none of the arbitrary constants can be made redundant.

- From this perspective, we can also conclude that

$$a_n = A2^n + B3^n$$

is the general solution of the second order recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = 0$$

because the solution contains $A$ and $B$ as the two independent arbitrary constants.
Recall

- A polynomial $f(\lambda)$ has $\lambda_0$ as one of its roots precisely when $f(\lambda_0) = 0$. 

For example, if $f(\lambda) = \lambda^2 - 5\lambda + 6$, then $\lambda_0 = 3$ is one of its roots because $f(\lambda_0) = f(3) = 3^2 - 5 \times 3 + 6 = 0$. 

In other words, 3 is a root of the equation $\lambda^2 - 5\lambda + 6 = 0$. 

In general, a polynomial equation of order $n$ will have exactly $n$ roots, some of which may be distinct while others may be repeated, some of which may be real while others may be complex numbers. 

A second order polynomial equation $a\lambda^2 + b\lambda + c = 0$, $a \neq 0$ has two roots, $\lambda_1$ and $\lambda_2$, given by 

$$\lambda_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \lambda_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a},$$

If $\lambda_1 = \lambda_2$, then the two roots are the repeated roots and $\lambda_1$, which is the same as $\lambda_2$, is a root of multiplicity 2. If $\sqrt{b^2 - 4ac} \geq 0$ the two roots are real numbers, else if $\sqrt{b^2 - 4ac} < 0$ the two roots are complex numbers.
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\lambda_1 = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a},
\lambda_2 = -\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a},
$$

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$$f(\lambda) = (\lambda - \lambda_0)^m g(\lambda)$$

such that $g(\lambda)$ is again a polynomial with $g(\lambda_0) \neq 0$. 

For example, the polynomial equation $\lambda^2 - 6\lambda + 9 = 0$ has a root 3 of multiplicity 2.

This is because $\lambda^2 - 6\lambda + 9 = (\lambda - 3)^2 \times 1$, in which $f(\lambda) = \lambda^2 - 6\lambda + 9$, $g(\lambda) = 1$, $\lambda_0 = 3$ and $m = 2$.

If we solve the equation $\lambda^2 - 6\lambda + 9 = 0$ through the use of the above root formula for $\lambda_1$ and $\lambda_2$, we see that both $\lambda_1$ and $\lambda_2$ are equal to the same value 3.

This also explains why 3 is a root of multiplicity 2 for the equation $\lambda^2 - 6\lambda + 9 = 0$.
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- A polynomial equation $f(\lambda) = 0$ has a root $\lambda_0$, i.e., $f(\lambda_0) = 0$, if and only if $f(\lambda) = (\lambda - \lambda_0) \cdot g(\lambda)$ for another nonzero polynomial $g(\lambda)$.

- If $\lambda_0$ is furthermore a root of multiplicity $m > 1$ of $f(\lambda) = 0$, then $\lambda_0$ must be a root of multiplicity $m - 1$ of $g(\lambda) = 0$.

- A root of multiplicity 1 is called a simple root.

- A simple root is thus not a repeated root.
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Simplest Case of General Solutions

Theorem

If the characteristic equation of an \( m \)-th order homogeneous, linear, constant coefficient recurrence relation

\[
c_m a_{n+m} + c_{m-1} a_{n+m-1} + \cdots + c_1 a_{n+1} + c_0 a_n = 0, \quad c_m c_0 \neq 0, \quad n \geq 0
\]

has \( m \) distinct roots \( \lambda_1, \lambda_2, \cdots, \lambda_m \), then

\[
a_n = A_1 \lambda_1^n + A_2 \lambda_2^n + \cdots + A_m \lambda_m^n
\]

with arbitrary constants \( A_1, \cdots, A_m \) is the general solution of the recurrence relation.
Proof.

First we show $a_n = \sum_{i=1}^{m} A_i \lambda_i^n$ is a solution. For this purpose we substitute the expression for $a_n$ into the recurrence relation and obtain

\[
c_m a_{n+m} + \cdots + c_1 a_{n+1} + c_0 a_0 = \\
= c_m (A_1 \lambda_1^{n+m} + \cdots + A_m \lambda_m^{n+m}) + \cdots + c_1 (A_1 \lambda_1^{n+1} + \cdots + A_m \lambda_m^{n+1}) \\
+ c_0 (A_1 \lambda_1^n + \cdots + A_m \lambda_m^n) \\
= A_1 (c_m \lambda_1^{n+m} + \cdots + c_1 \lambda_1^{n+1} + c_0 \lambda_1^n) + A_2 (c_m \lambda_2^{n+m} + \cdots + c_1 \lambda_2^{n+1} + c_0 \lambda_2^n) \\
+ \cdots + A_m (c_m \lambda_m^{n+m} + \cdots + c_1 \lambda_m^{n+1} + c_0 \lambda_m^n) \\
= \sum_{i=1}^{m} A_i \lambda_i^n (c_m \lambda_i^m + \cdots + c_1 \lambda_i + c_0) = 0.
\]

Hence $a_n = \sum_{i=1}^{m} A_i \lambda_i^n$ is a solution. Since the solution involves $m$ arbitrary constants $A_1, \cdots, A_m$, it is in fact a general solution.
Proof.

Alternatively, we can also argue that for any initial values of $a_0, \cdots, a_{m-1}$, since the linear system of $m$ equations

$$\sum_{i=1}^{m} A_i \lambda_i^k = a_k, \quad k = 0, 1, \cdots, m - 1$$

has a (unique) solution for $A_1, \cdots, A_m$, the expression $a_n = \sum_{i=1}^{m} A_i \lambda_i^n$ is indeed a general solution.

If the roots $\lambda_1, \cdots, \lambda_m$ are not distinct, i.e., there are repeated roots, then

$$a_n = \sum_{i=1}^{m} A_i \lambda_i^n$$

is still a solution but is not a general solution. The general solution for such cases will be dealt with in the next section. $\square$
Corollary (1)

Given two solutions \( \{x_n\} \) and \( \{y_n\} \) of an \( m \)-th order homogeneous, linear, constant coefficient recurrence relation, any linear combination of them

\[ z_n = Ax_n + By_n \]

where \( A, B \) are constants, is also a solution of the same recurrence relation.
**Simplest Case of General Solutions**

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Given two solutions \( \{x_n\} \) and \( \{y_n\} \) of an \( m \)-th order homogeneous, linear, constant coefficient recurrence relation, any linear combination of them \( z_n = Ax_n + By_n \) where \( A, B \) are constants, is also a solution of the same recurrence relation.

**Proof.**

Given \( c_m a_{n+m} + c_{m-1} a_{n+m-1} + \cdots + c_1 a_{n+1} + c_0 a_n = 0, \ c_m c_0 \neq 0, \ n \geq 0 \), if \( \{x_n\} \) and \( \{y_n\} \) are solutions, then

\[
\begin{align*}
c_m x_{n+m} + c_{m-1} x_{n+m-1} + \cdots + c_1 x_{n+1} + c_0 x_n &= 0 \quad | \cdot A \\
c_m y_{n+m} + c_{m-1} y_{n+m-1} + \cdots + c_1 y_{n+1} + c_0 y_n &= 0 \quad | \cdot B
\end{align*}
\]

If we multiply the first relation by \( A \), the second by \( B \) and then sum them up, we get

\[
c_m [Ax_{n+m} + By_{n+m}] + c_{m-1} [Ax_{n+m-1} + By_{n+m-1}] + \cdots + c_1 [Ax_{n+1} + By_{n+1}] + c_0 [Ax_n + By_n] = 0
\]

that is

\[
\begin{align*}
c_m z_{n+m} + c_{m-1} z_{n+m-1} + \cdots + c_1 z_{n+1} + c_0 z_n &= 0
\end{align*}
\]

which shows that \( z_n \) is a solution for the same recurrence relation.
Corollary (2)

If the recurrence relation is a non-homogeneous one, then the difference of any two solutions is a solution of the homogeneous version of the recurrence relation.

Proof.

Given \(c_m a^n + m + c_{m-1} a^{n-1} + \cdots + c_1 a^{n+1} + c_0 a^n = g(n), c_m c_0 \neq 0, n \geq 0\), if \(\{x_n\}\) and \(\{y_n\}\) are solutions, then they satisfy

\[
c_m x_n + m + c_{m-1} x_{n-1} + \cdots + c_1 x_{n+1} + c_0 x_n = g(n)\]
\[
c_m y_n + m + c_{m-1} y_{n-1} + \cdots + c_1 y_{n+1} + c_0 y_n = g(n)\]

and their difference gives

\[
c_m [x_n + m - y_n + m] + c_{m-1} [x_{n-1} + m - y_{n-1} + m] + \cdots + c_1 [x_{n+1} - y_{n+1}] + c_0 [x_n - y_n] = g(n) - g(n) = 0\]

\[
c_m z_n + m + c_{m-1} z_{n-1} + \cdots + c_1 z_{n+1} + c_0 z_n = 0\]
Simplest Case of General Solutions

**Corollary (2)**

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**Proof.**

Given \( c_m a_{n+m} + c_{m-1} a_{n+m-1} + \cdots + c_1 a_{n+1} + c_0 a_n = g(n) \), \( c_m c_0 \neq 0 \), \( n \geq 0 \), if \( \{x_n\} \) and \( \{y_n\} \) are solutions, then they satisfy

\[
c_m x_{n+m} + c_{m-1} x_{n+m-1} + \cdots + c_1 x_{n+1} + c_0 x_n = g(n)
\]

\[
c_m y_{n+m} + c_{m-1} y_{n+m-1} + \cdots + c_1 y_{n+1} + c_0 y_n = g(n)
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\[
c_m [x_{n+m} - y_{n+m}] + c_{m-1} [x_{n+m-1} - y_{n+m-1}] + \cdots +
\]

\[
+ c_1 [x_{n+1} - y_{n+1}] + c_0 [x_n - y_n] = g(n) - g(n) = 0
\]

\[
c_m z_{n+m} + c_{m-1} z_{n+m-1} + \cdots + c_1 z_{n+1} + c_0 z_n = 0
\]
Example

Find the general solution of

\[ a_{n+2} - 5a_{n+1} + 6a_n = 0, \quad n \geq 0. \]

Give also the particular solution satisfying \( a_0 = 0 \) and \( a_1 = 1 \).
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Find the general solution of

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Give also the particular solution satisfying \( a_0 = 0 \) and \( a_1 = 1 \).

Solution. Since the associated characteristic equation is

\[ \lambda^2 - 5\lambda + 6 = 0 \]

and has 2 distinct roots \( \lambda_1 = 2 \) and \( \lambda_2 = 3 \), the general solution for the recurrence relation, according to the theorem earlier on, is \( a_n = A_12^n + A_23^n \), \( n \geq 0 \), where \( A_1 \) and \( A_2 \) are 2 arbitrary constants. To find the particular solution, we need to determine \( A_1 \) and \( A_2 \) explicitly using the initial conditions \( a_0 = 0 \) and \( a_1 = 1 \). Hence we require

\[
\begin{align*}
    a_0 &= A_12^0 + A_23^0 = 0 \\
    a_1 &= A_12^1 + A_23^1 = 1
\end{align*}
\]

i.e.,

\[
\begin{align*}
    A_1 + A_2 &= 0 \\
    2A_1 + 3A_2 &= 1
\end{align*}
\]

which has the solution \( A_1 = -1, \ A_2 = 1 \) and, thus, the particular solution is \( a_n = -2^n + 3^n \) for \( n \geq 0 \).
Note

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- To check the above particular solution, we see

\[
\begin{align*}
 a_0 &= -20 + 30 = 0 \\
 a_1 &= -21 + 31 = 1 \\
 a_{n+2} - 5a_{n+1} + 6a_n &= (-2n^2 + 3n^2) - 5(-2n + 3n) + 6(-2n) \\
 &= -2n(2 - 5 	imes 2 + 6) \\
 &= -2n(0) = 0.
\end{align*}
\]

i.e., all conditions are satisfied.
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  \[ a_0 = -2^0 + 3^0 = 0 \]
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  $a_0 = -2^0 + 3^0 = 0$
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  $a_{n+2} - 5a_{n+1} + 6a_n = (-2^{n+2} + 3^{n+2}) - 5(-2^{n+1} + 3^{n+1}) + 6(-2^n + 3^n)$
  $= -2^n (2^2 - 5 \times 2 + 6) + 3^n (3^2 - 5 \times 3 + 6) = 0 \| 0$
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$$= -2^n (2^2 - 5 \times 2 + 6) + 3^n (3^2 - 5 \times 3 + 6) = 0,$$

i.e., all conditions are satisfied.